GALOIS THEORY 2019: HW 5 Feedback
For assessment: Problems 1, 2, 3
Due by noon Tuesday, week 11 of the term

There we a lot of very good papers, and then some showing some serious lack of understanding of important concepts. Many detailed comments are written on these papers; here I address more commen errors.

1. (a) With $\alpha \in \phi(H)$, $\sigma \in G$, one shows that for $\tau \in H$, $\tau(\sigma(\alpha)) = \sigma(\alpha)$. To conclude that $\sigma(\alpha) \in \phi(H)$, one needs to note that this holds **for all** $\tau \in H$.
   (b) or (c) To show $\sigma\phi(H) = \phi(H)$: replacing σ by σ^{-1} is (a) we get $\sigma^{-1}\phi(H) \subseteq \phi(H)$, so we have $\phi(H) \subseteq \sigma\phi(H)$. Since we also have $\sigma\phi(H) \subseteq \phi(H)$, we get $\sigma\phi(H) = \phi(H)$.

2. (a) With $\alpha = \sqrt[3]{7}$ and $\zeta = e^{2\pi i/3}$, we have that $\alpha, \alpha\zeta, \alpha\zeta^2$ are the roots of f. One shows that $f = m_\alpha(\mathbb{Q})$. So $[\mathbb{Q}(\alpha) : \mathbb{Q}] = 3$. To show $[\mathbb{Q}(\alpha, \zeta) : \mathbb{Q}] = 6$, it is not sufficient to just consider $m_\zeta(\mathbb{Q})$ and say that because this has degree 2, then $[\mathbb{Q}(\alpha, \zeta) : \mathbb{Q}(\alpha)] = 2$.
   (b) Let $L = \mathbb{Q}(\alpha, \zeta)$. Since we know (i) $G = Gal(L : \mathbb{Q})$ is isomorphic to a subgroup of S_3, (ii) every elements of G is determined by how it permutes the roots of f, and (iii) $|G| = |S_3|$, we can conclude that every permutation of the roots of f corresponds to an element of G. But in general, this is not true! So in this problem, either one has to make the oberservations (i), (ii), (iii), and then the conclusion as above, **or** one has to describe how to construct the elements of G.
   (c) Showing that, say, $\alpha \in \phi(H)$ for some subgroup H of G, one can conclude that $\mathbb{Q}(\alpha) \subseteq \phi(H)$. But this alone is not sufficient to show $\mathbb{Q}(\alpha) = \phi(H)$.