INTRODUCTION TO PROOFS: HOMEWORK ASSIGNMENTS

Your solutions should be organised to proceed logically, and should be written in complete sentences.

Homework Assignment 1 (from the posted Exercises for the course): Exercises 1.2; 1.5(a); 1.6 (shown below)

1.2. Let $X = \{x \in \mathbb{R} : \ x \neq 1 \}$, $Y = \{y \in \mathbb{R} : \ y \neq 3 \}$. Define $f : X \to Y$ by $f(x) = \frac{3x}{x-1}$. **Fact:** f does in fact map X into Y, and f is surjective.
  (a) Find a function $g : Y \to X$ so that $g \circ f$ is the identity function on X. (In your scratch work, to find g you may want to set $y = f(x)$ and solve for x in terms of y; however, in your solution, you should begin by defining g and then proceed to prove that for every $y \in Y$, we have $g(y) \in X$, and for every $x \in X$, we have $g \circ f(x) = x$.)
  (b) Show that g is surjective. (In your scratch work, you may want to begin by setting $x = g(y)$ and then solving for y. However, in your presentation, you should begin by choosing (arbitrary) $x \in X$, then simply produce the value for y and demonstrate that $y \in Y$ with $g(y) = x$.)
  (c) Show that $f \circ g$ is the identity map on Y.

1.5. (a) Suppose $f : X \to Y$, $g : Y \to Z$. Show that if f and g are surjective then so is $g \circ f$. (Begin by choosing [arbitrary] $z \in Z$. You must show there is some $x \in X$ so that $g \circ f(x) = z$. Use first that g is surjective.)

1.6. Suppose $f : X \to Y$, $g : Y \to X$ so that $g \circ f$ is the identity map on X (so for all $x \in X$, we have $g \circ f(x) = x$). Suppose f is surjective; prove that $f \circ g$ is the identity map on Y. (Suggestion: Take $x \in X$; evaluate $f \circ g \circ f(x)$ in two ways. Now take $y \in Y$; use that f is surjective and what you have just shown to conclude that $f \circ g(y) = y$.)