# INTRODUCTION TO PROOFS: HOMEWORK ASSIGNMENTS

**Your solutions should be organised to proceed logically, and should be written in complete sentences.**

**Homework Assignment 2 (from the posted Exercises for the course):** Exercises 1.7(a); 2.1(c); 2.2(b); 2.5(a); 3.2 (shown below)

1.7. Suppose $f : X \to Y$ is bijective.
  (a) Suppose $g : Y \to Z$ is bijective (and hence we know $g \circ f$ is bijective). Show that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ (here $(g \circ f)^{-1}$ denotes the inverse of $g \circ f$, which we have seen is unique). (So you need to show that for any $z \in Z$, we have $(g \circ f)^{-1}(z) = f^{-1} \circ g^{-1}(z)$. Take $y \in Y$ so that $g^{-1}(z) = y$, and take $x \in X$ so that $f^{-1}(y) = x$. Recall that we have a "recipe" for describing an inverse function.)

2.1. Suppose $P, Q, R$ are propositions. Show:
  (c) Show that $P \vee (Q \wedge R) \iff (P \vee Q) \wedge (P \vee R)$.

2.2 Suppose $P, Q$ are propositions. Show:
  (b) $\neg(P \implies Q) \iff (P \wedge \neg Q)$.

2.5. Suppose $P, Q, R$ are propositions. Show:
  (a) Show that $(P \implies Q) \iff R$ is not equivalent to $P \implies (Q \iff R)$.

2.6. We use $\mathbb{R}^2$ to denote $\mathbb{R} \times \mathbb{R}$, and $\mathbb{R}^3$ to denote $\mathbb{R} \times \mathbb{R} \times \mathbb{R}$. Define $f : \mathbb{R}^2 \to \mathbb{R}^3$ by

$$f((x, y)) = (x + y, x - y, x^2 + y^2).$$

  (a) Prove that $f$ is injective. (Suggestion: Suppose that $(x, y), (u, v) \in \mathbb{R}^2$ so that $f((x, y)) = f((u, v))$. Show that $(x, y) = (u, v)$. (So you must show that $x = u$ and $y = v$.)
  (b) Prove that $f$ is not surjective. (So you need to choose explicit values for $u, v, w$ and then deduce that for **any** choices for $x, y \in \mathbb{R}$, it is impossible to have $f((x, y)) = (u, v, w)$. Suggestion: For the sake of contradiction, suppose that $f$ is surjective. Carefully choose explicit values $u, v, w \in \mathbb{R}$, and suppose that $(x, y) \in \mathbb{R}^2$ with $f((x, y)) = (u, v, w)$; derive a contradiction.)

3.2. Negate the following propositions:
  (a) $\exists i \in I$ so that $x \in B_i$,
  (b) $\exists c \in \mathbb{R}$ so that $\forall \varepsilon > 0$, $\exists N \in \mathbb{Z}_+$ so that $\forall n \geq N$, $|a_n - c| < \varepsilon$.
  (c) $\forall \varepsilon > 0$, $\exists N \in \mathbb{Z}_+$ so that $\forall n \geq N$, $\forall m \geq N$, $|a_n - a_m| \leq \varepsilon$.
  (d) $\forall f : X \to Y$, $\forall A \subseteq X$, $\forall B \subseteq X$, $f(A \setminus B) \not\subseteq f(A) \setminus f(B)$.