INTRODUCTION TO PROOFS: HOMEWORK ASSIGNMENTS

Your solutions should be organised to proceed logically, and should be written in complete sentences.

Homework Assignment 3 (from the posted Exercises for the course):
Exercises 3.3(b); 3.5; 4.1(b); 4.3(a); 4.7(a) (shown below)

3.3. (b) Let $(0,1) = \{x \in \mathbb{R} : \ 0 < x < 1\ \}$. Define $f : (0,1) \to \mathbb{R}$ by $f(x) = \frac{1-x}{x}$. Show that f is injective.

3.5. Suppose $f : X \to Y$, $g : Y \to X$ so that $g \circ f$ is the identity map on X, meaning that for all $x \in X$, we have $g \circ f(x) = x$. Suppose g is injective; prove that $f \circ g$ is the identity map on Y. (Suggestion: Take $y \in Y$. Evaluate $g \circ f \circ g(y)$ in two ways, using that $(g \circ f) \circ g = g \circ f \circ g = g \circ (f \circ g)$; then use that g is injective. Recall that in the notes for this section, we presented the contrapositive of the definition of g being injective; this will be useful in this proof.) [Note: This is proved in the lecture notes by a different method.]

4.1. Suppose A, B, C are subsets of a set X.
(b) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. (Suggestion: Show that for $x \in X$, we have $x \in A \cup (B \cap C) \iff x \in (A \cup B) \cap (A \cup C)$.)

4.3. Suppose A, B are subsets of a set X.
(a) Prove that $(A \setminus B)^c = A^c \cup B$. (So you must show that $x \notin A \setminus B$ if and only if $x \notin A$ or $x \in B$).

4.7. (a) Suppose $f : X \to Y$, and $U \subseteq X$, $V_1, V_2 \subseteq Y$. Show that $f^{-1}(V_1 \cup V_2) = f^{-1}(V_1) \cup f^{-1}(V_2)$.