INTRODUCTION TO PROOFS: HOMEWORK ASSIGNMENTS

Your solutions should be organised to proceed logically, and should be written in complete sentences.

Homework Assignment 4 (from the posted Exercises for the course): Exercises 5.1(b)(i), (iii); 5.3; 5.5(b), (c); 6.2(b); 6.6; 6.7(b).

5.1. (b) Determine whether each of the following relations are reflexive, symmetric, transitive; justify your answers.

(i) Let $X = \{ f : \mathbb{R} \to \mathbb{R} \}$. Define a relation $\sim$ on X by $f \sim g$ if $f(0) = g(0)$.

(iii) Let Z be the collection of all subsets of $\mathbb{Q}$. Define a relation on Z by $A \sim B$ if $A \subseteq B$.

5.3. Let X be a set and $\sim$ a relation on X. Define

$$N = \{x \in X : \neg(x \sim x) \}.$$

Let

$$B = \{b \in X : (\forall n \in N)(b \sim n) \wedge (\forall n \notin N)[\neg(b \sim n)] \}.$$

Show that $B = \emptyset$. (Suggestion: Suppose there is some $b \in B$; show that $b \in N \implies b \notin N$, and $b \notin N \implies b \in N$. Then explain why it is impossible to have $b \in B$.)

5.5. (b) Find $x \in \mathbb{Z}$ so that $0 \leq x < 7$ and $x \equiv 5^{10} \pmod 7$.

(c) Find $x \in \mathbb{Z}$ so that $0 \leq x < 11$ and $x \equiv 3^5 + 8^4 \pmod{11}$.

6.2. (b) Suppose $a, b, c \in \mathbb{Z}$ so that $c \neq 0$, $c|ab$, and $\text{hcf}(b, c) = 1$. Show that $c|a$. (Suggestion: Use the fact that since $\text{hcf}(b, c) = 1$, $\exists s, t \in \mathbb{Z}$ so that $bs + ct = 1$, and that $a = 1 \cdot a$.)

6.6. (a) Find $s, t \in \mathbb{Z}$ so that $11s + 13t = 1$.

(b) Find $x \in \mathbb{Z}$ so that $x \equiv 2 \pmod{11} \wedge x \equiv 3 \pmod{13}$. (Suggestion: Use the algorithm presented in the proof of the Chinese Remainder Theorem.)

(c) Find $x \in \mathbb{Z}$ so that $x \equiv 4 \pmod{11} \wedge x \equiv 7 \pmod{13}$.

6.7. Use induction to prove the following identity.

(b) $\sum_{i=0}^{n} \frac{1}{2^i} = 2 - \frac{1}{2^n}$ for $n \in \mathbb{Z}$ with $n \geq 0$.