

INTRODUCTION TO PROOFS

Week 1 Tutorial Questions

1. Suppose X is a set.
 - (a) Is it always valid to write “Take $a \in X$ ”?
 - (b) Is it always valid to write “Suppose $a \in X$ ”?
 - (c) What is the difference between the meanings of the words “take” and “suppose”?

2. Given a subset X of \mathbb{Z} , we say that a number $y \in \mathbb{Z}$ is an *upper bound for X* if, for all $x \in X$, we have $x \leq y$.
Set
$$X = \{x \in \mathbb{Z} : x < 7\}.$$
 - (a) Is 10 an upper bound for X ?
 - (b) Is 7 an upper bound for X ?
 - (c) Is 7 the least, or the smallest, upper bound for X ? If not, is there a least upper bound for X ?
 - (d) Does X have a maximal, or largest, element?
 - (e) Does X have a lower bound in \mathbb{Z} , meaning a number $z \in \mathbb{Z}$ so that for all $x \in X$ we have $z \leq x$?

3. Given a subset A of \mathbb{R} , we say that a number $y \in \mathbb{R}$ is an *upper bound for A* if, for all $x \in A$, we have $x \leq y$.
Set
$$A = \{x \in \mathbb{R} : -1 < x < 5/2\}.$$
 - (a) Is 100 an upper bound for A ?
 - (b) Now we want to show that $5/2$ is a least, or smallest, upper bound for A .
 - (i) First, explain why $5/2$ is an upper bound for A .
 - (ii) Now suppose that $y \in \mathbb{R}$ with $y < 5/2$. Explain why y is **not** an upper bound for A (so you need to explain why there is some element $b \in A$ with $y < b$).
 - (c) Does A have a greatest, or largest, lower bound? Explain your answer.

4. Recall that \mathbb{Z}_+ is the set of *positive* integers. Discuss why any nonempty subset B of \mathbb{Z}_+ has a minimal, or smallest, element.