INTRODUCTION TO PROOFS

Week 11 Tutorial Questions

Present your answers in complete sentences.

1. (a) Let P, Q represent propositions (meaning statements that are either true or false, but not both simultaneously). Use a truth table to show that $\neg(P \implies Q)$ is equivalent to $P \wedge \neg Q$.

   (b) Recall that we found that an equivalent definition of $f : X \to Y$ being injective is:

   $$\forall x_1, x_2 \in X, \ (f(x_1) = f(x_2) \implies x_1 = x_2).$$

   Negate this statement, meaning find a statement that is logically equivalent to

   $$\neg[\forall x_1, x_2 \in X, \ (f(x_1) = f(x_2) \implies x_1 = x_2)].$$

   Write your answer without using the symbol $\neg$.

2. Define a map $f : \mathbb{Z}_+ \to \mathbb{Z}_+$ that is surjective but not injective, and demonstrate that this map is surjective but not injective.

3. Suppose A is a finite set with $|A| = n$ for some $n \in \mathbb{Z}$ with $n \geq 0$. Recall that $\mathcal{P}(A)$ denotes the poser set of A, meaning that

   $$\mathcal{P}(A) = \{C : \ C \subseteq A \}.$$

   Use induction to show that $|\mathcal{P}(A)| = 2^n$. (Suggestion: For the induction step, suppose A is a nonempty set, and fix an element $u \in A$. Let $B = A \smallsetminus \{u\}$. Argue that there is a bijection between $\{C : \ C \subseteq B \}$ and $\{D : \ D \subseteq A, \ u \in D \}$. Then show that this means that $\mathcal{P}(A) = 2\mathcal{P}(B)$, and use your induction hypothesis to conclude that $|\mathcal{P}(A)| = 2^{k+1}$ where $|A| = k + 1$.)