INTRODUCTION TO PROOFS

Week 3 Tutorial Questions

Present all your answers in complete sentences.

Recall: When we write $f : X \to Y$, we are indicating that f is a function from X to Y where X, Y are necessarily nonempty sets.
Also, a function $f : X \to Y$ is injective if, for any $x, x' \in X$ with $x \neq x'$, we have $f(x) \neq f(x')$.

1. Suppose $f : X \to Y$ is injective, and fix $y \in f(X)$.
   (a) Briefly explain why there exists some $x \in X$ so that $f(x) = y$.
   (b) Now suppose $x, x' \in X$ with $f(x) = y$ and $x \neq x'$. Briefly explain why $f(x') \neq y$. (Suggestion: begin by recalling the definition of injective.)
   (c) Using (a) and (b), briefly explain why there is a **unique** $x \in X$ so that $f(x) = y$. (Recall that f is injective and $y \in f(X)$.)

2. Suppose $f : X \to Y$, $g : Y \to Z$, $h : Z \to W$.
   (a) Suppose $s : X \to Z$. For $x \in X$, what is $h \circ s(x)$? (Your answer should be presented in a complete sentence, so it could start "We have $h \circ s(x) =$".)
   (b) Now suppose $s = g \circ f$ and $x \in X$. According to your answer in (a), what is $h \circ (g \circ f)(x)$? Now use the definition of $g \circ f$ to further evaluate $h \circ (g \circ f)(x)$.
   (c) Supppose $t : Y \to W$. For $x \in X$, what is $t \circ f(x)$?
   (d) Now suppose $t = h \circ g$, and $x \in X$. According to your answer in (c), what is $(h \circ g) \circ f(x)$? Now use the definition of $h \circ g$ to further evaluate $(h \circ g) \circ f(x)$.
   (e) What can you conclude from (b) and (c)?

3. Negate the following statements, concluding with statements that do not use the symbol $\neg$.
   (a) $\exists m \in \mathbb{Z}$ such that $|a_m| \geq 5$.
   (b) $\forall n \in \mathbb{Z}_+, n > m \implies |a_n| < 5$.
   (c) $(\exists m \in \mathbb{Z}$ such that $|a_m| \geq 5) \wedge (\forall n \in \mathbb{Z}_+, n > m \implies |a_n| < 5)$.

4. Let P and Q represent propositions (so P and Q each represent a statement that can either be true or false, but not both at once). Using a truth table, show that

$$[P \vee Q] \iff [\neg P \implies Q].$$