INTRODUCTION TO PROOFS

Week 7 Tutorial Questions

Note: comments in square brackets [such as this comment] are not necessary for a complete solution.

1. Assume X is a nonempty set with an equivalence relation $\sim$. For $x, y \in X$, let $[x]$ denote the equivalence class of x, and $[y]$ the equivalence class of y. Also suppose $z \in [x] \cap [y]$, and $w \in [y]$. Arrange the following phrases to produce a proof that $w \in [x]$. You may use phrases more than once (or not at all).
   (1) because $\sim$ is reflexive.
   (2) because $\sim$ is symmetric.
   (3) because $\sim$ is transitive.
   (4) by definition.
   (5) by assumption.
   (6) We have $y \sim z$
   (7) We have $w \sim x$
   (8) We have $w \sim y$
   (9) We have $w \sim z$
   (10) We have $x \sim z$
   (11) We have $z \sim x$ and $z \sim y$
   (12) We have $w \sim x$
   (13) We have $w \in [x]$

2. Suppose X is a set.
   (a) Suppose $A, D \subseteq X$. Show that $(A \cap D)^c = A^c \cup D^c$.
   (b) Suppose $\{B_i : i \in \mathbb{Z}_+\}$ is a collection (i.e. a set) of subsets of X. For $n \in \mathbb{Z}_+$, let $P(n)$ be the statement

   $$(B_1 \cap B_2 \cap \cdots \cap B_n)^c = B_1^c \cup B_2^c \cup \cdots \cup B_n^c.$$

   Use induction to prove that $P(n)$ holds for all $n \in \mathbb{Z}$ with $n \geq 2$. (Suggestion: for the induction step, make choices for A and D and use part (a).)