INTRODUCTION TO PROOFS
Week 9 Tutorial Questions

Present your answers in complete sentences.

Working through this tutorial sheet, for some questions you may want to use the following.

Fact 1. By definition, a (nonempty) set X is countable if there is a bijective function $f : \mathbb{Z}_+ \to X$.

Fact 2. Corollary 8.4 says the following. Suppose $X \subseteq \mathbb{Z}_+$. Then X is finite or countable.

1. Suppose X is a set.
   (a) Suppose $A, D \subseteq X$. Show that $(A \cap D)^c = A^c \cup D^c$.
   (b) Suppose $\{B_i : \ i \in \mathbb{Z}_+ \}$ is a collection (i.e. a set) of subsets of X. For $n \in \mathbb{Z}_+$, let $P(n)$ be the statement
   $$(B_1 \cap B_2 \cap \cdots \cap B_n)^c = B_1^c \cup B_2^c \cup \cdots \cup B_n^c.$$
   Use induction to prove that $P(n)$ holds for all $n \in \mathbb{Z}$ with $n \geq 2$. (Suggestion: for the induction step, make choices for A and D and use part (a).)

2. Suppose X is a countable set. Suppose A is a subset of X; prove that A is finite or countable. (Suggestion: If A is finite then we are done. So suppose A is infinite. Recall that since X is countable, there is a bijective map $f : X \to \mathbb{Z}_+$. Construct an injective map from A into $\mathbb{Z}_+$, and **prove** this map is injective.)

3. Let $A = \{x \in \mathbb{Z}_+ : \ x \text{ is even } \}$ and let $B = \{x \in \mathbb{Z}_+ : \ x \text{ is odd } \}$. We know that A and B are infinite subsets of $\mathbb{Z}_+$, and $\mathbb{Z}_+$ is countable; so by Corollary 8.4, A and B are countable. Now suppose that X is a countable set. Show that there is a subset C of X so that C and $X \setminus C$ are both countable. (Recall that a countable set is necessarily infinite. Suggestion: begin with the definition of X being countable, then use this to identify A with a countable subset of X.)