EXPLICIT MATRICES FOR HECKE OPERATORS ON SIEGEL MODULAR FORMS

Lynne H. Walling

Abstract. We present an explicit set of matrices giving the action of the Hecke operators $T(p), T_j(p^2)$ on Siegel modular forms.

Introduction

It is well-known that the space of elliptic modular forms of weight k has a basis of simultaneous eigenforms for the Hecke operators, and the Fourier coefficients of an eigenform (and hence the eigenform) are completely determined by its eigenvalues and first Fourier coefficient. In the theory of Siegel modular forms, the role of the Hecke operators is not yet completely understood, thus there are many avenues open for conjecture and exploration, including computational exploration. The purpose of this note is to present an explicit set of matrices giving the action of the Hecke operators on Siegel modular forms, with the goal of facilitating computational exploration. (This construction also yields an explicit set of matrices giving the action of Hecke operators on Jacobi modular forms; we remark on this further at the end of this note.)

Definitions and results

For F a Siegel modular form of degree n and p a prime, we define the Hecke operator $T(p)$ by

$$F|T(p) = p^{n(k-n-1)/2} \sum_M F| \begin{pmatrix} \frac{1}{p} I_n & \\ & I_n \end{pmatrix} M$$

where M runs over a complete set of coset representatives for $(\Gamma' \cap \Gamma)\backslash\Gamma$, with $\Gamma = Sp_n(\mathbb{Z})$ and

$$\Gamma' = \begin{pmatrix} pI_n & \\ & I_n \end{pmatrix} \Gamma \begin{pmatrix} \frac{1}{p} I_n & \\ & I_n \end{pmatrix}.$$

1991 *Mathematics Subject Classification.* 11F41.

Key words and phrases. Siegel modular forms; Hecke operators; Jacobi modular forms.

Similarly, for $1 \le j \le n$, we define $T_j(p^2)$ by

$$F|T_j(p^2) = \sum_M F| \begin{pmatrix} \frac{1}{p}I_j & & & \\ & I_{n-j} & & \\ & & pI_j & \\ & & & I_{n-j} \end{pmatrix} M$$

where M runs over a complete set of coset representatives for $(\Gamma'_j \cap \Gamma)\backslash\Gamma$; here

$$\Gamma'_j = \begin{pmatrix} pI_j & & & \\ & I_{n-j} & & \\ & & \frac{1}{p}I_j & \\ & & & I_{n-j} \end{pmatrix} \Gamma \begin{pmatrix} \frac{1}{p}I_j & & & \\ & I_{n-j} & & \\ & & pI_j & \\ & & & I_{n-j} \end{pmatrix}.$$

In [3], we determine the action of Hecke operators on Fourier coefficients of a Siegel modular form by first describing a complete set of coset representatives for the Hecke operators. We index the cosets using lattices; the coset representatives are then explicitly described except for a choice of $G \in GL_n(\mathbb{Z})$ associated to each lattice. There are infinitely many possible choices for each G; in this note we make an explicit choice for each G.

We first construct matrices for $T_j(p^2)$ (and for the averaged operators $\widetilde{T}_j(p^2)$ introduced in [3]); then we do the same for $T(p)$.

For $T_j(p^2)$, we construct these G as follows. For (nonnegative) integers r_0, r_2 with $r_0 + r_2 \le j$ and $r_1 = j - r_0 - r_2$, we call $\mathcal{P}$ a partition of type (r_0, r_2) for (n, j) if $\mathcal{P}$ is an ordered partition

$$(\{d_1, \dots, d_{r_0}\}, \{b_1, \dots, b_{r_1}\}, \{a_1, \dots, a_{r_2}\}, \{c_1, \cdots, c_{n-j}\})$$

of $\{1, 2, \dots, n\}$. (Note that if some $r_i = 0$ or $n - j = 0$, a set in the partition could be empty.) Given a partition $\mathcal{P}$ of type (r_0, r_2), we let $\mathcal{G}_{\mathcal{P}} \subseteq GL_n(\mathbb{Z})$ consist of all matrices $G = (G_0, G_1, G_2, G_3)$ constructed as follows. G_0 is the $n \times r_0$ matrix with ℓ, t-entry 1 if $\ell = d_t$, and 0 otherwise. G_1 is an $n \times r_1$ matrix with ℓ, t-entry $\beta_{\ell t}$ where $\beta_{\ell t} = 1$ if $\ell = b_t$, $\beta_{\ell t} = 0$ if $\ell < b_t$ or $\ell = a_i$ (some i) or $\ell = b_i$ (some $i \ne t$), and otherwise $\beta_{\ell t} \in \{0, 1, \dots, p-1\}$. G'_2 is an $n \times r_2$ matrix with ℓ, t-entry $\alpha_{\ell t}$ where $\alpha_{\ell t} = 1$ if $\ell = a_t$, $\alpha_{\ell t} = 0$ if $\ell < a_t$ or $\ell = a_i$ (some $i \ne t$), and otherwise $\alpha_{\ell t} \in \{0, 1, \dots, p-1\}$. G''_2 is an $n \times r_2$ matrix with ℓ, t-entry $\delta_{\ell t}$ where $\delta_{\ell t} = 0$ if $\ell \ne d_i$ (any i), and otherwise $\delta_{\ell t} \in \{0, 1, \dots, p-1\}$. $G_2 = G'_2 + pG''_2$. G_3 is an $n \times (n-j)$ matrix with ℓ, t-entry $\gamma_{\ell t}$ where $\gamma_{\ell t} = 1$ if $\ell = c_t$, $\gamma_{\ell t} = 0$ if $\ell < c_t$ or $\ell = a_i$ or b_i (some i) or $\ell = c_i$ (some $i \ne t$), and otherwise $\gamma_{\ell t} \in \{0, 1, \dots, p-1\}$.

Note that (G_0, G_1, G'_2, G_3) is a (column) permutation of an integral lower triangular matrix with 1's on the diagonal, and thus is an element of $GL_n(\mathbb{Z})$. Also, it is easy to see that there is an elementary matrix E so that

$$(G_0, G_1, G'_2, G_3)E = (G_0, G_1, G'_2 + pG''_2, G_3) = G,$$

and so $G \in GL_n(\mathbb{Z})$. (After proving Theorem 1, we describe G^{-1} as a product of four explicit matrices.)

We let $\mathcal{G}_{r_0,r_2} = \cup_{\mathcal{P}} \mathcal{G}_{\mathcal{P}}$ where $\mathcal{P}$ varies over all partitions of type (r_0, r_2). We set

$$D_{r_0,r_2} = \begin{pmatrix} I_{r_0} &&& \\ & pI_{r_1} && \\ && p^2 I_{r_2} & \\ &&& I_{n-j} \end{pmatrix}.$$

Also, we let $\mathcal{Y}_{r_0,r_2}$ be the set of all (integral) matrices of the form

$$\begin{pmatrix} Y_0 & Y_2 & 0 & Y_3 \\ p\ {}^tY_2 & Y_1 && \\ 0 &&& \\ {}^tY_3 &&& \end{pmatrix}$$

where Y_0 is symmetric, $r_0 \times r_0$, with entries varying modulo p^2, Y_1 is symmetric, $r_1 \times r_1$, with entries varying modulo p, Y_2 is $r_0 \times r_1$ with entries varying modulo p, and Y_3 is $r_0 \times (n-j)$ with entries varying modulo p. We let $\mathcal{Y}'_{r_0,r_2}$ be those matrices in $\mathcal{Y}_{r_0,y_2}$ that satisfy the additional condition $p \nmid \det Y_1$ (which is trivially satisfied if $r_1 = 0$).

Theorem 1. *Given a degree n Siegel modular form F, $1 \le j \le n$,*

$$F|T_j(p^2) = \sum_M F| \begin{pmatrix} \frac{1}{p}I_j &&& \\ & I_{n-j} && \\ && pI_j & \\ &&& I_{n-j} \end{pmatrix} M$$

where

$$M = \begin{pmatrix} D & Y \\ & D^{-1} \end{pmatrix} \begin{pmatrix} G^{-1} & \\ & {}^tG \end{pmatrix}$$

varies so that for some r_0, r_2 with $r_0 + r_2 \le j$, $D = D_{r_0,r_2}$, $Y \in \mathcal{Y}'_{r_0,r_2}$, and $G \in \mathcal{G}_{r_0,r_2}$. Also, with

$$\widetilde{T}_j(p^2) = p^{j(k-n-1)} \sum_{0 \le t \le j} \begin{bmatrix} n-t \\ j-t \end{bmatrix}_p T_t(p^2)$$

where $\begin{bmatrix} m \\ r \end{bmatrix}_p = \prod_{i=0}^{r-1} \frac{p^{m-i}-1}{p^{r-i}-1}$,

$$F|\widetilde{T}_j(p^2) = p^{j(k-n-1)} \sum_M F| \begin{pmatrix} \frac{1}{p}I_j &&& \\ & I_{n-j} && \\ && pI_j & \\ &&& I_{n-j} \end{pmatrix} M$$

where

$$M = \begin{pmatrix} D & Y \\ & D^{-1} \end{pmatrix} \begin{pmatrix} G^{-1} & \\ & {}^tG \end{pmatrix}$$

varies so that for some r_0, r_2 with $r_0 + r_2 \le j$, $D = D_{r_0,r_2}$, $Y \in \mathcal{Y}_{r_0,r_2}$, and $G \in \mathcal{G}_{r_0,r_2}$.

Proof. As mentioned above, in Proposition 2.1 of [3] we found a complete set of coset representatives indexed by lattices (Ω, Λ_1) where, for Λ a fixed lattice of rank n, Ω varies subject to $p\Lambda \subseteq \Omega \subseteq \frac{1}{p}\Lambda$, and $\overline{\Lambda}_1$ varies over all codimension $n-j$ subspaces of $\Lambda \cap \Omega / p(\Lambda + \Omega)$. With $\{\Lambda : \Omega\}$ denoting the invariant factors of Ω in Λ and r_0 the multiplicity of the invariant factor p, r_2 the multiplicity of the invariant factor $\frac{1}{p}$, the action of the coset representatives corresponding to (Ω, Λ_1) is given by the matrices

$$\begin{pmatrix} D & Y \\ & D^{-1} \end{pmatrix} \begin{pmatrix} G^{-1} & \\ & {}^tG \end{pmatrix}$$

where $D = D_{r_0,r_2}$, Y varies over $\mathcal{Y}'_{r_0,r_2}$, and $G = G(\Omega, \Lambda_1)$ is any change of basis matrix so that, relative to a fixed basis $(x_1, \dots, x_n)$ for Λ,

$$\Omega = \Lambda G D^{-1} \begin{pmatrix} pI_j & \\ & I_{n-j} \end{pmatrix}, \ \Lambda_1 = \Lambda G \begin{pmatrix} 0_{r_0} & & \\ & I_{r_1} & \\ & & 0 \end{pmatrix}$$

where $r_1 = j - r_0 - r_2$. (Thus only those Ω occur where $r_0 + r_2 \le j$.) So we need to show that such a pair (Ω, Λ_1) corresponds to a unique $G \in \mathcal{G}_{r_0,r_2}$. We do this by constructing all (Ω, Λ_1) for each pair of parameters (r_0, r_2), simultaneously building a partition $\mathcal{P}$ and making choices of $\alpha_{\ell t}, \beta_{\ell t}, \gamma_{\ell t}, \delta_{\ell t}$ so that with G constructed according to our recipe above, we can take G for $G(\Omega, \Lambda_1)$.

Notice that when $p\Lambda \subseteq \Omega \subseteq \frac{1}{p}\Lambda$, the Invariant Factor Theorem (81:11 of [5]) tells us we have compatible decompositions:

$$\Lambda = \Lambda_0 \oplus \Lambda'_1 \oplus \Lambda_2,$$
$$\Omega = p\Lambda_0 \oplus \Lambda'_1 \oplus \frac{1}{p}\Lambda_2.$$

On the other hand, given Λ, such an Ω is determined by $\Omega' = \Lambda_2 + p\Lambda$ and $(p\Lambda'_1 \oplus \Lambda_2) + p\Omega'$. Also, in $\Lambda \cap \Omega / p(\Lambda + \Omega)$, $\overline{\Lambda}_2 = 0$, so Λ_1 can be chosen so that in $\Omega'/p\Omega'$, $\overline{\Lambda}_1 \subseteq \overline{p\Lambda'_1} \subseteq \overline{p\Lambda}$.

So to begin our construction of Ω, Λ_1 and $G = G(\Omega, \Lambda_1)$, in $\Lambda/p\Lambda$ we choose a dimension r_2 subspace $\overline{C}'$; let $(\overline{v}'_1, \dots, \overline{v}'_{r_2})$ be a basis for $\overline{C}'$. Each $\overline{v}'_t$ is a linear combination over $\mathbb{Z}/p\mathbb{Z}$ of the $\overline{x}_i$; by adjusting the $\overline{v}'_t$ we can assume

$$\overline{v}'_t = \overline{x}_{a_t} + \sum_{\ell > a_t} \overline{\alpha}_{\ell t} \overline{x}_\ell$$

where $a_1, \dots, a_{r_2}$ and distinct and $\overline{\alpha}_{\ell t} = 0$ if $\ell = a_i$ (some $i \ne t$). Let $\alpha_{\ell t} \in \{0, 1, \dots, p-1\}$ be a preimage of $\overline{\alpha}_{\ell t}$.

Now let Ω' be the preimage in Λ of $\overline{C}'$. In $\Omega'/p\Omega'$ we will construct a dimension $n - r_0$ subspace $\overline{C}$ so that $\dim(\overline{C} \cap \overline{p\Lambda}) = n - r_0 - r_2$, distiguishing a dimension r_1

subspace $\overline{p\Lambda_1}$ of $\overline{C} \cap \overline{p\Lambda}$. We begin by choosing $\overline{p\Lambda_1}$ to be a dimension r_1 subspace of $\overline{p\Lambda}$; let $\overline{pu_1}, \dots, \overline{pu_{r_1}}$ be a basis for $\overline{p\Lambda_1}$. Since $\overline{px_{a_i}} = 0$ in $\Omega'/p\Omega'$, we can adjust the $\overline{pu_t}$ so that

$$\overline{pu_t} = \overline{px_{b_t}} + \sum_{\ell > b_t} \overline{\beta}_{\ell t} \overline{px_\ell}$$

where $b_1, \dots, b_{r_1}$ are distinct, $b_t \neq a_i$ (any i), and $\overline{\beta}_{\ell t} = 0$ if $\ell = a_i$ (some i) or $\ell = b_i$ (some $i \neq t$). Let $\beta_{\ell t} \in \{0, 1, \dots, p-1\}$ be a preimage of $\overline{\beta}_{\ell t}$.

Now extend $\overline{p\Lambda_1}$ to a dimension $n - r_0 - r_2$ subspace $\overline{p\Lambda_1'}$ of $\overline{p\Lambda}$ in $\Omega'/p\Omega'$. Extend $(\overline{pu_1}, \dots, \overline{pu_{r_1}})$ to a basis

$$(\overline{pu_1}, \dots, \overline{pu_{r_1}}, \overline{pw_1}, \dots, \overline{pw_{n-j}})$$

for $\overline{p\Lambda_1'}$ so that

$$\overline{pw_t} = \overline{px_{c_t}} + \sum_{\ell > c_t} \overline{\gamma}_{\ell t} \overline{px_\ell},$$

where $c_1, \dots, c_{n-j}$ are distinct, $c_t \neq a_i, b_i$ (any i), and $\overline{\gamma}_{\ell t} = 0$ if $\ell = a_i$ (some i), or $\ell = b_i$ (some $i \neq t$). Let $\gamma_{\ell t} \in \{0, 1, \dots, p-1\}$ be a preimage of $\overline{\gamma}_{\ell t}$.

Now we extend $\overline{p\Lambda_1'}$ to a dimension $n - r_0$ space $\overline{C}$ so that the dimension of $\overline{C} \cap \overline{p\Lambda}$ is $n - r_0 - r_2 = r_1 + n - j$, and we extend $(\overline{pu_1}, \dots, \overline{pw_1}, \dots)$ to a basis

$$(\overline{pu_1}, \dots, \overline{pu_{r_1}}, \overline{pw_1}, \dots, \overline{pw_{n-j}}, \overline{pv_1}, \dots, \overline{pv_{r_2}})$$

for $\overline{C}$. Taking $d_1, \dots, d_{r_0}$ so that

$$(\{d_1, \dots, d_{r_0}\}, \{b_1, \dots, b_{r_1}\}, \{a_1, \dots, a_{r_2}\}, \{c_1, \dots, c_{n-j}\})$$

is a partition of $\{1, \dots, n\}$, we can take

$$\overline{v}_t = \overline{v}_t' + \sum_{m=1}^{r_0} \overline{\delta}_{mt} \overline{px_{d_m}}$$

for some $\overline{\delta}_{mt}$; let $\delta_{mt} \in \{0, 1, \dots, p-1\}$ be a preimage of $\overline{\delta}_{mt}$.

Now let $p\Omega$ be the preimage in Ω' of $\overline{C}$. So with

$$\begin{aligned} u_t &= x_{b_t} + \sum_{\ell > b_t} \beta_{\ell t} x_\ell \ (1 \leq t \leq r_1), \\ v_t &= x_{a_t} + \sum_{\ell > a_t} \alpha_{\ell t} x_\ell + p \sum_m \delta_{mt} x_{d_m} \ (1 \leq t \leq r_2), \\ w_t &= x_{c_t} + \sum_{\ell > c_t} \gamma_{\ell t} x_\ell \ (1 \leq t \leq n - j), \end{aligned}$$

the vectors

$$(px_{d_1}, \dots, px_{d_{r_0}}, u_1, \dots, u_{r_1}, \frac{1}{p} v_1, \dots, \frac{1}{p} v_{r_2}, w_1, \dots, w_{n-j})$$

form a basis for Ω, and $(\overline{u}_1, \dots, \overline{u}_{r_1})$ is a basis for $\overline{\Lambda}_1$ in $\Lambda \cap \Omega / p(\Lambda + \Omega)$. $\square$

Remark. Given $G \in \mathcal{G}_{\mathcal{P}}$ as above, $G^{-1} = E_1E_2E_3E_4$ where the E_i are $n \times n$ matrices constructed as follows. E_1 has i, i-entry 1 $(1 \le i \le n)$; for $1 \le \ell \le r_0$, E_1 has $\ell, r_0 + t$-entry $-\beta_{d_\ell t}$ $(1 \le t \le r_1)$, $\ell, r_0 + r_1 + t$-entry $-\alpha_{d_\ell t} - p\delta_{d_\ell t}$ $(1 \le t \le r_2)$, $\ell, r_0 + r_1 + r_2 + t$-entry $-\gamma_{d_\ell t}$ $(1 \le t \le n - j)$, and all other entries 0. So for $1 \le t \le n - j$, column $r_0 + r_1 + r_2 + t$ of GE_1 has a 1 in row c_t, and zeros elsewhere. E_2 has i, i-entry 1 $(1 \le i \le n)$; for $1 \le \ell \le n - j$, E_2 has $r_0 + r_1 + r_2 + \ell, r_0 + t$-entry $-\beta_{c_\ell t}$ $(1 \le t \le r_1)$, $r_0 + r_1 + r_2 + \ell, r_0 + r_1 + t$-entry $-\alpha_{c_\ell t}$ $(1 \le t \le r_2)$, and zeros elsewhere. Thus for $1 \le t \le r_1$, column $r_0 + t$ of GE_1E_2 has a 1 in row b_t, and zeros elsewhere. E_3 has i, i-entry 1 $(1 \le i \le n)$; for $1 \le \ell \le r_1$, $1 \le t \le r_2$, E_3 has $r_0 + \ell, r_0 + r_1 + t$-entry $-\alpha_{b_\ell t}$ and zeros elsewhere. Thus $GE_1E_2E_3 = {}^tE_4$ is a permutation matrix; E_4 has 1 as its ℓ, d_ℓ-entry $(1 \le \ell \le r_0)$, 1 as its $r_0 + \ell, b_\ell$-entry $(1 \le \ell \le r_1)$, 1 as its $r_0 + r_1 + \ell, a_\ell$-entry $(1 \le \ell \le r_2)$, 1 as its $r_0 + r_1 + r_2 + \ell, c_\ell$-entry $(1 \le \ell \le n - j)$, and zeros elsewhere. So $GE_1E_2E_3E_4 = I$.

We follow a similar procedure to construct matrices for $T(p)$: For $0 \le r \le n$, we let $\mathcal{G}_r$ be the set of matrices G so that for some ordered partition $\mathcal{P} = (\{d_1, \dots, d_r\}, \{a_1, \dots, a_{n-r}\})$ of $\{1, 2, \dots, n\}$, for $1 \le t \le r$, column t of G has 1 in row d_t and zeros elsewhere, and for $1 \le t \le n - r$, the $\ell, r + t$-entry of G is $\alpha_{\ell t}$ where $\alpha_{\ell t}$ is 1 if $\ell = a_t$, $\alpha_{\ell t} = 0$ if $\ell < a_t$ or $\ell = a_i$ (some $i \ne t$), and otherwise $\alpha_{\ell t} \in \{0, 1, \dots, p - 1\}$. (So $G \in GL_n(\mathbb{Z})$ with $G^{-1} = E_1E_2$ where E_1 has i, i-entry 1 $(1 \le i \le n)$, $\ell, r + t$-entry $-\alpha_{d_\ell t}$, with zeros elsewhere, and E_2 has t, d_t-entry 1 for $1 \le t \le r$, $r + t, a_t$-entry 1 for $1 \le t \le n - r$, and zeros elsewhere.) Let $\mathcal{Y}_r$ be the collection of matrices $\begin{pmatrix} Y & \\ & 0 \end{pmatrix}$ where Y varies over integral $r \times r$, symmetric matrices modulo p, and let $D_r = \begin{pmatrix} I_r & \\ & pI_{n-r} \end{pmatrix}$.

Theorem 2. *Given a degree n Siegel modular form F,*

$$F|T(p) = p^{n(k-n-1)/2} \sum_M F| \begin{pmatrix} \frac{1}{p} I_n & \\ d & I_n \end{pmatrix} M$$

where

$$M = \begin{pmatrix} D & Y \\ & D^{-1} \end{pmatrix} \begin{pmatrix} G^{-1} & \\ & {}^tG \end{pmatrix}$$

varies so that for some r, $0 \le r \le n$, $D = D_r$, $Y \in \mathcal{Y}_r$, and $G \in \mathcal{G}_r$.

Proof. Using Proposition 3.1 of [3], we only need to show that as G varies over $\mathcal{G}_r$, $\Omega = \Lambda G p D_r^{-1}$ varies once over all lattices Ω where $p\Lambda \subseteq \Omega \subseteq \Lambda$, $[\Lambda : \Omega] = p^r$. So, similar to the proof of Theorem 1, we construct all the Ω as well as a specific basis for each Ω.

Let $\overline{C}$ be a dimension $n - r$ subspace of $\Lambda / p\Lambda$. Choose a basis $\overline{v}_1, \dots, \overline{v}_{n-r}$ so that

$$\overline{v}_t = \overline{x}_{a_t} + \sum_{\ell > a_t} \overline{\alpha}_{\ell t} \overline{x}_\ell$$

where $a_1, \dots, a_{n-r}$ are distinct, $\overline{\alpha}_{\ell t} = 0$ if $\ell = a_i$ (some $i \neq t$); for each $\overline{\alpha}_{\ell t}$, take a preimage $\alpha_{\ell t} \in \{0, 1, \dots, p-1\}$. Then with $(\{d_1, \dots, d_r\}, \{a_1, \dots, a+n-r\})$ an ordered partition of $\{1, 2, \dots, n\}$ and G constructed according to our recipe preceding Theorem 2, we have $\Omega = \Lambda G p D_r^{-1}$. □

Remark. In [6] we discuss how a particular subgroup of $Sp_n(\mathbb{Z})$ acts on Jacobi forms on $f : \mathbb{H}_{(n-m)} \times \mathbb{C}^{n-m,m} \to \mathbb{C}$. From this we see that for $1 \leq j \leq n-m$,

$$f|T_j(p^2) = \sum_M f| \begin{pmatrix} \frac{1}{p} I_j & & & \\ & I_{n-j} & & \\ & & pI_j & \\ & & & I_{n-j} \end{pmatrix} M$$

where

$$M = \begin{pmatrix} D & Y \\ & D^{-1} \end{pmatrix} \begin{pmatrix} G^{-1} & \\ & {}^tG \end{pmatrix}$$

varies so that for some r_0, r_2 with $r_0 + r_2 \leq j$, $D = D_{r_0, r_2}$, $Y \in \mathcal{Y}'_{r_0, r_2}$, and $G \in \mathcal{G}_{\mathcal{P}}$ where $\mathcal{P}$ is a partition of type (r_0, r_2) so that $\{n-m+1, \dots, n\} \subseteq \{c_1, \dots, c_{n-j}\}$ (with the c_i arranged in ascending order). When $n - m < j \leq n$, the operator $T_j(p^2)$ changes the index, as does $T(p)$, and the matrices giving the action of these operators is a bit more complicated; explicit matrices for these operators are given in [6].

References

1. A.N. Andrianov, *Quadratic Forms and Hecke Operators*, Grundlehren Math. Wiss., Vol. 286, Springer-Verlag, 1987.
2. E. Freitag, *Siegelsche Modulfunktionen*, Grundlehren Math. Wiss., Vol. 254, Springer-Verlag, 1983.
3. J.L. Hafner, L.H. Walling, *Explicit action of Hecke operators on Siegel modular forms*, J. Number Theory **93** (2002), 34-57.
4. H. Maass, *Die Primzahlen in der Theorie der Siegelschen Modulformen*, Math. Ann. **124** (1951), 87-122.
5. O.T. O'Meara, *Introduction to Quadratic Forms*, Springer-Verlag, 1973.
6. L.H. Walling, *Restricting Hecke-Siegel operators to Jacobi modular forms*, (preprint).

L.H. Walling, Department of Mathematics, University of Bristol, Bristol BS8 1TW, England

E-mail address: `l.walling@bristol.ac.uk`