Analysis Workshop
Dirichlet's Drawer Principle and The Decimal Numbers

Story. In Alexandrian times (336-323 BC), and perhaps earlier, Greek mathematicians used what is called a *nonpositional* decimal system, using the Greek alphabet. For instance, ι was used to represent the number we now commonly write as 10, and δ was used to represent the number we commonly write as 4; but in this period, $\iota\delta$ and $\delta\iota$ were both used to represent the number we commonly write as 14. This system was used by the Greek mathematicians for (at least) 1500 years, and some scholars have argued that this method of representing numbers was detrimental to the growth of Greek algebra.

Contrastingly, the Hindu-Arabic numeral system is a *positional* decimal numeral system, that was invented by Indian mathematicians between the 1st and 4th century AD. By the 9th century AD, the system was adopted in Arabic mathematics, and many centuries later was spread to Europe. This is essentially the system we use today in Europe (and elsewhere) to represent numbers (although in different regions, different "glyphs" were used to represent the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9).

It is believed by scholars that Chinese numeration has always been decimal, and it is clear that the ancient Chinese "rod numeral" system was a positional decimal numeral system. They used rods (made from various materials, such as bamboo or bones) to represent the digits 1, 2, 3, 4, 5, 6, 7, 8, 9; before having a notation for the digit 0, they would leave a blank space in their positional numeral system. This numeral system was stablised during the Han Dynasty (206 BC-220 AD), but due to lack of translations, this system was long unfamiliar to people outside China.

Here you complete the first steps toward showing that we can find some $n \in \mathbb{N}$ so that the first two digits after the dot in the decimal expansion of $n\sqrt{2}$ are whatever we want them to be. In this investigation, you should make use of the *Dirichlet drawer principle*, also called the *pigeonhole principle*, which states: given n objects (or pigeons) and m drawers (or pigeonholes), after placing each object in a drawer, if $n > m$ then at least one drawer must contain more than one object. (Depicted below are 10 pigeons in 9 pigeonholes.)

For further exploration:

(a) Before going into battle, the Chinese military strategist Sun Tzu (also spelled Sunzi, c. 544-496 BC, and author of *The Art of War*) used counting rods to carry out calculations to determine winning strategies. *The Art of War* has been studied for many years by military theorists around the world, and in recent decades it has become popular among the general populous, particularly among those in the business sector.

(b) From the website

https://study.com/academy/lesson/native-american-mathematics-history-mathematicians.html

"Native American societies generally used either base-ten or base-20 counting systems, and recorded numerical data through notches in wood, woven chords, and painted bark, among other lightweight and transportable systems. Some even simply used their fingers. Think that sounds a bit primitive? By using different combinations of fingers and a complex understanding of multiplication, some Native American cultures could count sequentially up to 1,000 using their ten fingers alone."

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