FEEDBACK: ASSESSED HW2

Overall, people did well on this homework.

1.
   - To show that $L : K$ is algebraic, you need to show that for **any** $\alpha \in L$, we have that α is algebraic over K (it's not enough to show that $L = K(\alpha_1, \ldots, \alpha_n)$ with $\alpha_1, \ldots, \alpha_n$ algebraic over K).
   - To construct L as $K(\alpha, \ldots, \alpha_n)$ via an inductive process, set $m = [L : K]$. If $L = K$ then $L = K(1)$. So suppose $L \neq K$. Choose $\alpha_1 \in L \setminus K$; argue that $[K(\alpha_1) : K] < \infty$, and if $[K(\alpha_1) : K] = m$ then you are done. Otherwise, choose $\alpha_2 \in L \backslash K(\alpha_2)$, not just $\alpha_2 \neq \alpha_1$. (A priori, $K(\alpha_1)$ has infinitely many elements α_2, and for each such α_2 we have $K(\alpha_1, \alpha_2) = K(\alpha_1)$. So the algorithm/induction may not terminate if you take $\alpha_i \neq \alpha_1, \ldots, \alpha_{i-1}$.)
   - If $\alpha \in L$ is the root of some $f \in K[t]$ this does not mean that $[K(\alpha) : K] = \deg f$ **unless** f is irreducible over K.
   - When you say that some element is algebraic, yo need to say over what field it is algebraic.
4.
   - There is no need to separate the cases $\alpha \in K$ and $\alpha \in L \setminus K$, as the argument for $\alpha \in L \setminus K$ also works for $\alpha \in K$.
   - Some explanations were not clear, sometimes because it was not made clear what was being assumed.
   - When $f \in K[t] \setminus K$, $\alpha \in L$ so that $f(\alpha) = 0$, then $f = (t - \alpha)g$ for some $g \in L[t]$, **not** $g \in K[t]$.
6. To begin, there are two approaches: (1) choose **nonconstant** $g \in \psi(L)[t]$, or (2) argue that (the extension of) ψ maps $L[t]$ onto $\psi(L)[t]$. In either case, one needs that any **nonconstant** element of $\psi(L)[t]$ is the image of a **nonconstant** element of $L[t]$. (Recall that the definition of an algebraically closed field is in terms of nonconstant polynomials over the field.)