**Quiz 2, 2019**

1. Let $P$ and $Q$ be statements. Which of the following are true statements? Select **all** that apply.
   (a) If $P \iff Q$, then proving that $Q$ is a true statement proves that $P$ is a true statement.
   (b) If $\neg Q \implies \neg P$ is a true statement, then $P \implies Q$ is a true statement.
   (c) If $P \implies Q$ is a true statement, then $P$ cannot be a false statement.
   (d) If $P \implies Q$ is a false statement, then $\neg P \vee Q$ is a false statement.

   [Answers: (a), (b), (d). Suggestion: review the truth table for $P \implies Q$, Theorem 2.1, and the meaning of the symbol $\iff$.]

2. Negate the following statement:

$$\forall x \in \mathbb{R}, (x \notin \mathbb{Z}_+) \implies [(\forall n \in \mathbb{Z}_+,\ x \neq 2n) \wedge (\forall n \in \mathbb{Z}_+,\ x \neq 2n+1)].$$

   (a) $\exists x \in \mathbb{R}$ such that $[(\exists n \in \mathbb{Z}_+$ such that $x = 2n) \vee (\exists n \in \mathbb{Z}_+$ such that $x = 2n+1)] \wedge (x \notin \mathbb{Z}_+)$.
   (b) $\exists x \in \mathbb{R}$ such that $[(\forall n \in \mathbb{Z}_+,\ x \neq 2n) \wedge (\exists n \in \mathbb{Z}_+$ such that $x = 2n+1)] \wedge (x \notin \mathbb{Z}_+)$.
   (c) $\forall x \in \mathbb{R}$, $[(\exists n \in \mathbb{Z}_+$ such that $x = 2n) \vee (\exists n \in \mathbb{Z}_+$ such that $x = 2n+1)] \wedge (x \notin \mathbb{Z}_+)$.

   [Answer: (a). Suggestion: review examples in Section 3 of the notes; remember that $[\neg(P \implies Q)] \iff [P \wedge \neg Q]$.]

3. To begin proving a function $f : X \to Y$ is surjective, you do which of the following?
   (a) Choose $x \in X$.
   (b) Choose $y \in Y$.
   (c) Choose $x_1, x_2 \in X$ and suppose $x_1 \neq x_2$.
   (d) Choose $y_1, y_2 \in Y$ and suppose $y_1 \neq y_2$.

   [Answer: (b). Suggestion: review the definition of surjective.]

4. Suppose $f : X \to Y$. According to the Propositions and Theorems in Section 1 of the posted and printed lecture notes, identify all of the statements below that are true.
   (a) If $f$ is surjective then $f$ has an inverse $f^{-1} : Y \to X$.
   (b) If $f$ is injective then $f$ has an inverse $f^{-1} : Y \to X$.
   (c) If $f$ has an inverse then its inverse is unique.
   (d) If $f$ has an inverse $f^{-1}$ and $x \in X$, then $(f \circ f^{-1})f(x) = f(x)$.

   [Answers: (c), (d). Suggestion: review the definition of the inverse of a function and Theorem 1.8.]

5. Suppose $A, B, C$ are nonempty subsets of a nonempty set $X$, and suppose $x \in A \setminus (B \setminus C)$. Identify all of the statements below that are correct.
   (a) We have $(x \in A) \wedge (x \notin B) \wedge (x \in C)$.

(b) We have $[(x \in A) \wedge (x \notin B)] \vee (x \in C)$.
(c) We have $(x \in A) \wedge [(x \notin B) \vee (x \in C)]$.

[Answer: (c). We have

$$\begin{aligned} x \in A \smallsetminus (B \smallsetminus C) &\iff x \in A \wedge \neg(x \in B \smallsetminus C) \\ &\iff x \in A \wedge \neg(x \in B \wedge x \notin C) \\ &\iff x \in A \wedge (x \notin B \vee x \in C).] \end{aligned}$$

6. Let $X = \{x \in \mathbb{R} : \ x > 5\ \}$, $Y = \{y \in \mathbb{R} : \ y > 2\ \}$. For $x \in X$, define $f(x) = \dfrac{2x}{x-5}$. We claim this defines a function $f$ that maps $X$ into $Y$. Which of the proofs below correctly shows this?
   (a) Suppose $x \in X$ and $\dfrac{2x}{x-5} > 2$. Since $x > 5$, we have $x - 5 > 0$ so $2x > 2(x-5)$. This implies $0 > -10$, which is true. Hence $f$ maps $X$ into $Y$.
   (b) Suppose $x \in X$. Thus $x > 5$, and so $x - 5 > 0$. We know $0 > -10$, so $2x > 2x - 10$. Hence $2x > 2(x-5)$, and since $x - 5 > 0$, we have $\dfrac{2x}{x-5} > 2$. Hence $f$ maps $X$ into $Y$.

   [Answer: (b). In your scratch work, you may want to work "backwards", determining how the result you want to deduce leads to the given hypothesis, and then try to reverse the argument. Then for your proof, you begin with the hypothesis and deduce the desired conclusion. (So (a) is like your scratch work, and (b) is a proper proof.)]