Possible Student Projects

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Point-set topology: A topology on a set is a system of "open" subsets, satisfying some simple axioms (such as the union or intersection of two open sets is again open). We can recast notions from analysis using topology; for instance, with X, Y topological spaces and $f : X \to Y$, f is continuous if and only if the inverse of any open subset of Y is an open subset of X. There are many things to explore in point-set topology. One possibility is the topic of topological groups; a group G is a topological group if G is equipped with a topology so that the map from $G \times G$ to G defined by $(x, y) \mapsto x \cdot y$ is continuous, and the map from G to G defined by $x \mapsto x^{-1}$ is continuous. So a topological group is equipped with a rich structure, providing interesting questions to explore. (Suitable for 10 or 20 cp; suitable as a group project.) Prerequisite: Algebra 2 or Linear Algebra 2 or Metric Spaces. Suggested texts: "Introduction to Topology" by Munkres; "General Topology" by Kelley.

Quadratic forms: A quadratic form on a vector space V captures notions of distance and orthogonality. For instance, with $V = \mathbb{R}^n$, the dot product gives us a quadratic form. One can also restrict a quadratic form to a "lattice" (for instance $\mathbb{Z}^n$ is a lattice in $\mathbb{R}^n$). There are many questions to explore with regard to the structure of a vector space or lattice equipped with a quadratic form. (Suitable for 10 or 20 cp, and possibly as a group project.) Prerequisite: Algebra 2 or Group Theory. Suggested texts: "Basic Quadratic Forms" by Gerstein; "Introduction to Quadratic Froms over Fields" by Lam.

Counting lattice points in an ellipse: Given an ellipse in the Cartesian plane, the number of lattice points (i.e. points in the plane with integer coefficients) contained in the ellipse is roughly equal to the area of the ellipse. So the question remains, what it the difference between the number of lattice points in the ellipse and the area of the ellipse? How does this difference change under changes to the ellipse that do not change the area? How does this difference vary as the ellipse is dilated? One can explore this problem initially using the computer. (Suitable for 10 cp.) Prerequisite: Some programming skills.

Other projects: If a student has a clear idea for a project, I am quite willing to discuss the project with them to determine whether I would be a suitable supervisor. Email l.walling@bristol.ac.uk.