

A Festival remembering Vic Snaith: Topology, Number Theory and interactions

Location: G.10 Lecture Theatre, Fry Building, University of Bristol, Woodland Road, BS8 1UG

Titles and abstracts: see below

Programme start: Monday July 10th 9:00am. Registration from 8:30.

Programme end: Final talk ends 13:00 Friday July 14th, Lunch ends 14:30 Friday

Monday	Tuesday	Wednesday	Thursday	Friday
9:00 Welcome				
9:15 Martin Taylor	9:15 Bruno Kahn	9:15 Hélène Esnault	9:15 Grzegorz Banaszak	9:15 Peter Symonds
10:15 Coffee	10:15 Coffee	10:15 Coffee	10:15 Coffee	10:15 Coffee
10:45 Wajid Mannan	10:45 George Pappas	10:45 Ferdinando Zanchetta	10:45 Lightning talks	10:45 Dustin Clausen
11:45 Break	11:45 Break	11:45 Break	11:45 Break	11:45 Break
12:00 Charanya Ravi	12:00 Inna Zakharevich	12:00 John Greenlees	12:00 posters	12:00 Ted Chinburg
13:00 Lunch	13:00 Lunch	13:00 Lunch	13:00 Lunch	13:00 Lunch
14:30 Rick Jardine (online)	14:30 Kirsten Wickelgren (online)	14:30 Local excursion	14:30 Vesna Stojanoska	
15:30 Coffee	15:30 Coffee		15:30 Coffee	
16:00 Jim Arthur (online)	16:00 Remembering Vic Snaith		16:00 Jenny Wilson (online)	
17:00 Wine Reception, chess, etc	Music, chess...			
	19:00 Conference dinner: Royal West of England Academy			

Titles and abstracts:

Jim Arthur

Title: Motives and automorphic representations

Abstract: Motives represent fundamental building blocks for number theory and geometry. They were postulated by Grothendieck almost sixty years ago, but there is still no general proof of their existence. Automorphic representations are spectral objects, with the analytic power one might hope to apply to motives. They were introduced in the form we now know them by Langlands, also almost sixty years ago. In this talk, we shall discuss work in progress to understand some remarkable new relationships that seem to be at the heart these objects.

Grzegorz Banaszak

Title: Algebraic K-theory and Arithmetic

Abstract: In my talk I will present relations between K-theory of number fields and classical conjectures in arithmetic: Kummer-Vandiver, Iwasawa, Leopold, Quillen-Lichtenbaum (now theorem) etc... I will discuss canonical subgroups of even K-groups of number fields, namely the groups of divisible elements, and I will show that all these conjectures can be expressed via the structure of these subgroups. Despite very close similarity of class groups and groups of divisible elements, I will show some advantages of working with groups divisible elements instead of working with class groups. The presentation will contain former results as well as recent results.

Ted Chinburg

Title: Explicit approximations of class field towers

Abstract: I will speak on joint work with Frauke Bleher on explicit constructions of infinite families \mathcal{F} of number fields K having small root discriminants $|d_K|^{1/[K:\mathbb{Q}]}$. By an explicit construction of \mathcal{F} we mean there is an algorithm that produces for each K in \mathcal{F} an explicit list of polynomials in $\mathbb{Q}[y]$ in time bounded by a polynomial in $\log[K : \mathbb{Q}]$ such that the roots of these polynomials generate K . The number fields K in a class field tower have constant $|d_K|^{1/[K:\mathbb{Q}]}$, but we do not know an explicit construction of an infinite set of fields in a class field tower. We give for each $\epsilon > 0$ an explicit construction of a family \mathcal{F} of K such that $|d_K|^{1/[K:\mathbb{Q}]}$ is $O([K : \mathbb{Q}]^\epsilon)$. By contrast, cyclotomic fields have discriminants larger than $O([K : \mathbb{Q}]^{1-\epsilon})$. To make our construction unconditional we use recent work of Matchett Wood and collaborators on Cebotarev density theorems as well as the Deligne-Serre theorem on modular forms of weight 1.

Dustin Clausen

Title: A modified Hodge conjecture

Abstract: The Hodge conjecture gives a criterion for a cohomology class on a projective complex manifold to come from an algebraic cycle. I will spend most of the time giving an introduction to the Hodge conjecture. Then I'll try to explain some reasons why the Hodge conjecture is hard to prove, and finally I'll propose a modification which should be easier to prove. This is based on joint work with Peter Scholze.

Hélène Esnault

Title: Integrality and obstruction for a finitely presented group to come from geometry

Abstract: We'll review how to use arithmetic methods based on the Langlands program to produce a new obstruction. The talk shall be based on joint work with Michael Groechenig for one part and Johan de Jong for the other.

John Greenlees

Title: Rational equivariant cohomology theories

Abstract: For the purpose of the talk, equivariant cohomology theories are invariants of topological spaces with an action of a group G . Many examples are well known: the theory of equivariant bundles gives us (topological) K -theories of various sorts, and the theory of manifolds gives us bordism of various sorts,

The talk will concentrate on the case that G is a compact Lie group and the cohomology theories take values in rational vector spaces. For many groups (and conjecturally for all) such cohomology theories can be described purely algebraically and perhaps classified. The talk will describe what is known and what the algebraic models look like, in a mixture of homological algebra and the point set topology of the space of subgroups of G .

Rick Jardine

Title: Local data structures

Abstract: Local data structures are systems of neighbourhoods within data sets. Specifications of neighbourhoods can arise in multiple ways, for example, from global geometric structure (stellar charts), combinatorial structure (weighted graphs), desired computational outcomes (natural language processing), or sampling. These examples are discussed, in the context of a theory of neighbourhoods.

This theory is a step towards a topological understanding of clustering for large data sets. Clusters for such data sets can only be approximated in practice, but approximations can be constructed from neighbourhoods via patching arguments that are derived from the recent Healy-McInnes UMAP construction. The patching arguments are enabled by changing the theoretical basis for data set structure, from metric spaces to extended pseudo metric spaces.

Bruno Kahn

Title: The rank spectral sequence on Quillen's Q -construction

Wajid Mannan

Title: Calculations in the K -theory of cyclotomic fields

Abstract: I will discuss the work of Vic Snaith, Zacky Choo, Ruben Sanchez-Garcia and myself, undertaken under Vic's grant "Computer assisted calculations of the borel regulator" in the mid 2000's. We were seeking to construct explicit elements in the odd K -theory of cyclotomic fields, and apply Borel's regulator map to them, in an effort to better understand the more mysterious even K -theory of cyclotomic fields.

George Pappas

Title: Galois structure of arithmetic torsors II

Charanya Ravi

Title: Equivariant localization theorem

Abstract: The Atiyah-Bott localisation theorem says that the equivariant cohomology of a space can be recovered, up to inverting some elements, from the equivariant cohomology of the fixed

point subspace. We discuss a categorification of this result in the algebraic geometric setting which allows us to deduce the theorem for all oriented theories (cohomology and Borel-Moore homology). This is based on a recent joint work with Adeel Khan.

Vesna Stojanoska

Title: Invertible Objects in Homotopy Theory

Abstract: Every time we have a notion of multiplication, such as the usual one of numbers or tensor product in a symmetric monoidal category, we can study objects which are invertible with respect to it, gaining structural insight. In number theory, Dirichlet's unit theorem determines the invertible elements in the ring O_K of integers of a number field K , while the ideal class group of K can be identified with the Picard group of invertible modules over O_K . The same concept but one categorical level up gives the Brauer group.

Notions such as these are also crucial in homotopy theory, and while generally it's difficult to compute any particular interesting example, keeping track of higher coherences results in better-behaved etale and Galois descent which helps with calculations. I will give a range of examples from my work with Antieau, Meier, and Mathew, computing the Picard, and Brauer groups of topological modular forms.

Peter Symonds

Title: Group actions on rings

Abstract: We consider a group (probably finite) acting on a ring (probably graded commutative and finitely generated), for example a polynomial ring. We can consider this as a representation of the group and try to understand it as such. Even better, we would like to understand how the geometry of the action of the group on the underlying variety of the ring influences this representation.

I will survey what is known and describe some recent results.

Martin Taylor

Title: Galois structure of arithmetic torsors I

Kirsten Wickelgren

Title: A quadratically enriched zeta function

Abstract: André Weil proposed a beautiful connection between algebraic topology and the number of solutions to equations over finite fields in a celebrated paper from 1948: the zeta function of a variety over a finite field is simultaneously a generating function for the number of solutions to its defining equations and a product of characteristic polynomials of endomorphisms of cohomology groups. The ranks of these cohomology groups are the Betti numbers of the associated complex manifold. We enrich the logarithmic derivative of the zeta function to a power series with coefficients in the Grothendieck–Witt group of stable isomorphism classes of unimodular modular forms, using traces of powers of Frobenius in A1-homotopy theory. We show the quadratically enriched logarithmic zeta function to be connected to the Betti numbers of the associated real manifold. This is joint work with Margaret Bilu, Wei Ho, Padma Srinivasan, and Isabel Vogt.

Jenny Wilson

Title: Towards conjectures of Rognes and Church–Farb–Putman

Abstract: In this talk I will give an overview of two related research programs. The first concerns the high-degree rational cohomology of the special linear group of a number ring R . Church-Farb-Putman conjectured that, when R is the integers, these cohomology groups vanish in a range close to their virtual cohomological dimension. I will discuss current progress on the Church-Farb-Putman conjecture, using the topology of certain associated simplicial complexes. The second project concerns Rognes' connectivity conjecture on a family of simplicial complexes (the "common basis complexes") arising in the study of algebraic K-theory. I will describe recent work proving Rognes' conjecture for fields, and the conjecture's connections to the cohomology of $SL_n(R)$. This talk includes results joint with Benjamin Brück, Jeremy Miller, Peter Patzt, and Robin Sroka.

Inna Zakharevich

Title: Doing algebra with geometry; a devissage story

Abstract: Quillen's original introduction of the K-theory of exact categories was remarkable not only for the progress it represented in the understanding of K-theory, but also because of the discovery of two vitally important theorems for the K-theory of abelian categories: localization and devissage. Localization shows how to construct a fiber sequence (leading to a long exact sequence in K-groups) associated to a "cofiber sequence" of abelian categories. It has a complicated statement and a complicated proof, but turns out to hold in almost every other K-theoretic context beyond abelian categories. Devissage, on the other hand, is much simpler to state: it states that if every object in a category is "constructed" out of objects in a given subcategory, then their K-theories are the same. This idea makes sense: K-theory splits objects into their component parts, so if the component parts are contained in the subcategory, surely all of the K-theoretic data is, as well. Amazingly, devissage turns out to be far more complicated to generalize, and is not known to hold in many contexts. In this talk we will discuss how to "combinatorialize" the algebraic ingredients that Quillen needs for his devissage result, and prove devissage for "ACGW-categories": categories that behave like abelian categories, but are constructed out of combinatorial, rather than algebraic, data. These include categories such as finite sets, graphs, and reduced schemes of finite type.

Ferdinando Zanchetta

Title: Operations in Algebraic K-theory

Abstract: Operations such as exterior power operations provide additional structure on K-theory that has been very useful for many purposes (e.g. in Riemann-Roch theory). In the context of higher algebraic K-theory of schemes, many a priori different definitions of them have been given over the last decades. After an introduction to the topic, I will explain the main ideas and techniques behind some of these constructions ending up with an explanation of how it is possible to show that they agree under reasonably general assumptions.

Lightning talk session (3 minute talks): Thursday 10:45-11:45

Names and titles:

Peng Du: Isotropic points in the Balmer spectrum of stable motivic homotopy categories

Biman Roy: \mathbb{A}^1 -homotopy theory and characterisation of the affine plane

Jason Semeraro: Explicit Brauer Induction for Spetses

David Solomon: Breaking Bad: SICs and Stark Units in Towers

Pratiksha Shingavekar: 3-Selmer groups, ideal class groups and the rational cube sum problem

Anja Meyer: On the Cohomology of Congruence Subgroups

Denver-James Logan Marchment: The action of the special linear group over a finite field on the Drinfeld curve

Jane Turner: Exterior power operations and Grayson's conjectural algebraic presentation of the relative K-groups

Makoto Yamagata: Diagonalizability and triangularizability of the matrices associated to maps between classifying spaces of connected compact Lie groups

Ming Ng: Adelic Geometry via Topos Theory

John Nicholson: Simple homotopy types of 4-manifolds

Michael Montoro: Connections between notions in the field of 3-manifolds and algebraic number theory

Andrei Konovalov: Topological K-theory of dg-categories and the lattice conjecture