Discussion on the paper by Andrieu, Doucet and Holenstein

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I offer my thanks to the authors for an inspirational paper. Their approach to constructing extended target distributions is powerful, can be exploited further and applied elsewhere. A key ingredient is the elucidation of the probability model underlying a SMC algorithm and the genealogical tree structures it generates. Two further developments on this theme are described below.

Firstly, at the end of one Conditional SMC run in the PG algorithm, the authors suggest sampling K from its full conditional under $\tilde{\pi}^N$, then deterministically tracing back the ancestral lineage of X_T^K, to yield

$$X_{1:T}^K := (X_1^{B_1^K}, X_2^{B_2^K}, ..., X_T^{B_T^K}). \tag{1}$$

There is an alternative. Having sampled K, for $n = T-1, ..., 1$ one could sample from

$$\tilde{\pi}^N(b_n^k|\bar{\mathbf{x}}_1, ..., \bar{\mathbf{x}}_n, \mathbf{a}_1, ..., \mathbf{a}_{n-1}, x_{n+1:T}^k, b_{n+1:T}^k, \theta),$$

with $X_{1:T}^K$ defined as before according to (1), but with newly-sampled ancestor indices.

The advantage of this "backward" sampling is that it enables exploration of all possible ancestral lineages and not only those obtained during the "forward" SMC run. This offers a chance to circumvent the path degeneracy phenomenon and obtain a faster mixing PG kernel, albeit at a slightly increased computational cost.

When $p_\theta(x_{1:T}, y_{1:T})$ arises from a state space model, it is straight-forward to verify that

$$\tilde{\pi}^N(b_n^k|\bar{\mathbf{x}}_1, ..., \bar{\mathbf{x}}_n, \mathbf{a}_1, ..., \mathbf{a}_{n-1}, x_{n+1:T}^k, b_{n+1:T}^k, \theta) \propto w_n^{b_n^k} f(x_{n+1}^{b_{n+1}^k}|x_n^{b_n^k}),$$

which uses the importance weights obtained during the forward SMC run. In this case the above procedure coincides with one draw using the smoothing method of [2].

Secondly, I believe that the PMCMC framework can be adapted to accommodate the particle filter of [1], which is somewhat different from the SMC algorithm considered in the present paper. Due to constraints on space I provide no specifics here, but I believe that suitable formulation of the probability model underlying the algorithm of [1] allows it to be manipulated as part of a PMCMC algorithm.

References

[1] P. Fearnhead and P. Clifford. On-line inference for hidden Markov models via particle filters. *Journal of the Royal Statistical Society B*, 65(4):887–899, November 2003.

[2] S. J. Godsill, A. Doucet, and M. West. Monte Carlo smoothing for nonlinear time series. *Journal of the American Statistical Association*, 99(465):156–168, March 2004.