Axiomatic Set Theory (MATH M1300)

Administrative Information

1. **Unit number and title:** MATH M1300 Axiomatic Set Theory ; **Level:** 4/M ; **Credit point value:** 20 credit points; **First Given:** 2004
2. **Lecturer/organiser:** Prof. P. Welch. Office Hours: Thursdays 9.00am Fry 2.26
   Course site see: http://www.people.maths.bris.ac.uk/~mapdw/teaching.html
3. **Prerequisites:** M32000 Set Theory and M30100 Logic are formal prerequisites, but please see me if you have not taken these courses.
4. **Examined in:** December
5. Three lectures per week in Weeks 1-5,7-11, Revision and Problems classes Week 6 ; Week 12: Revision classes.

The course site (see above) is the primary source of information.

Past Exam papers for the last three years will be posted on Blackboard. (But it is a policy that Solutions will *not* be posted – as the best kind of final revision you can do is to try and work through the past Exam papers.)

Please go to the Course Site (see above) for announcements, or any information relevant to the course.

Homework: this will be set, mostly weekly, but always on Fridays, with a deadline of 12.00pm the following Friday (see the course website for this) and will be marked by myself. Please note, the deadline has to be strict.

Unit aims

To develop the theory of Gödel's universe of constructible sets; to use this model to prove the consistency of various statements of mathematics with the currently accepted axioms of set theory.

General Description of the Unit

It is known that various straightforward mathematical statements are neither provable nor disprovable in the best available axiomatic system of set theory that we have. This system, Zermelo-Fraenkel set theory ("ZF"), provides a theoretical underpinning of all of mathematics, in that any mathematical statement, if provable, can be proven in this system. However certain straightforward statements, e.g., the Axiom of Choice (in one form: "every set can be wellordered") can be neither proved nor disproved in ZFC. Another is the Continuum Hypothesis ("CH": that every uncountable set of real numbers can be put in (1-1) correspondence with the set of all real numbers). The course will contain a discussion of the nature of axiomatic systems, the nature of concepts such as "provability", "unprovability" in such systems, and the status of Gödel's famous Incompleteness Theorems (roughly that any axiom system T extending that of, eg, Peano's system for arithmetic cannot prove a statement Con(T) encapsulating the consistency of that formal system.) in the setting of set theory.

There will follow an introduction to the axiomatics of ZF together with the construction of "L", a universe of sets invented by Gödel, This allowed him to show that both AC and CH were not disprovable.

If time permits we shall sketch Cohen's 1963 forcing method that showed how the CH was not provable from ZF; or else we may discuss further strong axioms of infinity, or ``large cardinals''

Relation to Other Units

This is the only unit which develops further the concepts in the Level 3 units Logic and Set Theory. It is particularly pertinent to those interest in, or taking courses in mathematics and philosophy.

Teaching Methods Lectures. Note that Course materials will be available on Blackboard, where there is a student Discussion Forum.

Learning Objectives

After taking this unit, students should:

1. Be familiar with the axiomatic basis of the theory of sets.
2. Be able to understand the notion of an "inner model" of set theory.
3. Be able to understand how such models enable consistency statements.
4. Have a working knowledge of the constructible hierarchy.

Assessment Methods

Examination. Four questions set in an 2 and ½ hour closed book exam. Answers to all 4 questions will be used for assessment. Calculators are not allowed for this examination.

(Homework does not count towards the final mark.)

Award of Credit Points

To gain credit points for this unit, students must gain a pass mark (50 or over) for the unit.

Texts A full text will be handed out, and reference to text books will not be needed. Alternative & Further Reading

Devlin, K. *Constructibility* Springer Perspectives in Mathematical Logic.

Drake, F. *Set Theory* North-Holland Series in Studies in Logic.

Drake, F & Singh, D. *Intermediate Set Theory* Wiley.

Kunen, K. *Set Theory: an Introduction to Independence Proofs.* Two Editions: North-Holland, and College Publications: Studies in Logic Series, vol.34. The latter volume has the advantage of being extremely cheap, c. £15, and gives a really good introduction to Cohen's theory of forcing (but is admittedly less good on inner models such as the constructible sets).

Schindler, R-D, *Set Theory* Universitext Springer. This is an excellent book for going on to more advanced topics, but is not particularly recommended for this course itself.

Syllabus

The Axioms of Zermelo-Fraenkel Set Theory with Choice; Class terms, relativisations to models; absoluteness ; Consistency proofs, reflection theorem ; closed and unbounded sets, stationary sets; regular and singular cardinals, cofinality; inaccessible cardinals; Gödel's Def function, and the definition of the constructible hierarchy L; the consistency of AC and GCH with ZFC .