Set Theory (MATH 32000)

Administrative Information

1. **Unit number and title:** MATH 32000 Set Theory
2. **Level:** H/6
3. **Credit point value:** 20 credit points
4. **First Given in this form:** 1996-99.
5. **Unit Organiser/Lecturer:** Prof. Philip Welch Office Hours: Fridays 10.00am Fry 2.26
   Course Site: www.people.maths.bris.ac.uk/~mapdw/teaching/ **This is the primary source of information for the course, and for Exercises Solutions.**
   Prerequisites: None, but this is a prerequisite for the Level M/7 unit Axiomatic Set Theory (MATH M1300).
   Examined: December
6. Three lectures per week in Weeks 1-5,7-11: Week 6 Problems classes and revision; 12: Three revision classes during the lecture slots
7. Student Support Sessions: This will be run by Sumedha Rathnayake bt22384@bristol.ac.uk in Weeks 2-12, Fridays 12.00-13.00. Fry G.14.

Past Exam papers for the last three years will be posted on Blackboard. (But it is a policy that Solutions will *not* be posted – as the best kind of final revision you can do is to try and work through the past Exam papers.)

Note that you are allowed to take in one page (two sides) of handwritten notes to the Examination. Accordingly the Examination questions will reflect this fact.

Please go to the Course Site (see above) for announcements, handouts or any information relevant to the course.

Lectures and Problems classes will be available through Blackboard

----

Unit aims
To introduce the students to the general theory of sets, as a foundational and as an axiomatic theory.

General Description of the Unit

The aim is to make the course of general interest to students who are not planning to specialize in mathematical logic or the Level M/7 Axiomatic Set Theory, but of special interest to those who are.

Set Theory can be regarded as a foundation for all, or most, of mathematics, in that any mathematical concept can be formulated as being about sets. The course shows how we can represent the natural numbers as sets and how principles such as proof by mathematical induction can be seen as being built up from very primitive notions about sets.

We shall see how the pitfalls of the various early "set theoretic paradoxes" such as that of Russell ("the set of all sets that do not contain themselves") were avoided. We develop Cantor's theory of transfinite ordinal numbers and their arithmetic through the introduction of his most substantial contribution to mathematics: the notion of wellordering. We shall see how an "arithmetic of the infinite" can be developed that extends naturally the arithmetic of the finite we all know. We shall introduce the principle of ordinal induction and recursion along the ordinals to extend that of mathematical induction and recursion along the natural numbers. Cantor's famous proof of the uncountability of the real continuum by a diagonal argument, and his revolutionary discovery that there were different "orders of infinity" - indeed infinitely many such - will feature prominently in our basic study of infinite cardinal numbers and their arithmetic.

We shall see how axiom sets can be used to develop this theory, and indeed the whole cumulative hierarchy of sets of mathematical discourse. There will be discussion of the axioms system ZF developed by Zermelo and Fraenkel in the wake of Cantor's work, and about the role the Axiom of Choice plays in set theory.

Relation to Other Units

Set Theory may be regarded as the foundation for all mathematics. This course is a prerequisite, is a for the level M/7 unit Axiomatic Set Theory M1300.

For students interested in the philosophy of mathematics: this course is related to a number of units in the philosophy department in philosophy of mathematics. It should thus be of interest to any joint Maths/Philosophy degree students, and to those on the MA in Logic and Philosophy of Mathematics.

Teaching Methods

Lectures and Exercise Sheets

Learning Objectives

The student should come away from this course with a basic understanding of such topics as the theory of partial orderings and well orderings, cardinality, ordinal numbers, and the role of the Axiom of Choice. He or she should also have become aware of the role of set theory as a foundation for mathematics, and of the part that axiomatic set theory has to play.

Assessment Methods

The assessment mark for Set Theory is calculated from a 2 ½-hour written examination in January, consisting of FOUR questions. A candidate answers to all questions will be used for assessment.

Raw scores on the examinations will be determined according to the marking scheme written on the examination paper. The marking scheme, indicating the maximum score per question, is a guide to the relative weighting of the questions. Raw scores are moderated as described in the Undergraduate Handbook.

If you fail this unit and are required to resit, reassessment is by a written examination in the August/September Resit and Supplementary exam period.

Award of Credit Points

Credit points will be awarded if a student passes the unit; i.e. attains a final mark of 40 or more.

Texts

Full Lecture notes will be provided. There are 3 copies of both [1] and [3] in the Queen's Building library (one of each on restricted Short Loan as is a copy of [2]).

[1] Elements of Set Theory, by H.Enderton, Academic Press.

[2] Classic Set Theory, D. Goldrei, Chapman & Hall.

[3] The theory of sets and transfinite ordinal numbers, by B. Rotman & G.T. Kneebone, Oldbourne Mathematical Series. QA248 ROT.

The following cover more than we need (they are also in the library):

[4] The Joy of Sets K. Devlin, Springer (at least two library copies)

In [5] (Chapters 1-5, 10,11 are sufficient) but is couched in more sophisticated language.

[5] Discovering Modern Set Theory I by W.Just and M. Weese, AMS Graduate Studies in Mathematics, Vol. 8.

Chapter I of the following has much to recommend it.

[6] K Kunen, The Foundations of Mathematics, College Publications Studies in Logic vol. 19.

If you are intending to do the 4'th Year Axiomatic Set Theory, the following is paperback and is a worthwhile purchase; for the M32000 course Chapters 1-7 cover what we need. [8] (Chapters I,II) has just appeared and is preferable in several ways.

[7] Intermediate Set Theory, by F.R.Drake and D. Singh, J.Wiley & Co.

[8] K. Kunen, Set Theory, College Publications Studies in Logic vol.34.

Syllabus

The basic principles and definitions of set theory.

The axiomatic definition of natural number.

Well-ordering and the Axiom of Choice.

Definition by transfinite induction

Ordinal and cardinal numbers.

The cumulative hierarchy of sets

Axiomatic Set Theory.