BCCS 2008/09: Graphical models and complex stochastic systems: Exercises 4

1. Consider the DAG with nodes labelled $\{1, 2, 3, \dots, 9\}$ and arrows $\{1 \to 4, 2 \to 3, 2 \to 4, 2 \to 5, 3 \to 5, 4 \to 6, 5 \to 7, 6 \to 8, 7 \to 8, 7 \to 9\}$. Use the global Markov property for DAGs to decide:

   (a) for which variables i is $x_1 \perp\!\!\!\perp x_i \mid x_8$?

   (b) for which variables i is $x_1 \perp\!\!\!\perp x_i \mid x_9$?

   (c) for which variables i is $x_1 \perp\!\!\!\perp x_i$?

2. Explain why the global Markov property for DAGs implies the local Markov property – no need to write it out in detail, an informal verbal explanation is plenty.

3. Consider the 'infant cardiac surgery' example from lecture 5. Suppose that hospitals A, B and C were all small local hospitals, and the rest are large teaching hospitals. What differences, if any, would you want to make to the model we used?

4. Studies are carried out in J different countries to assess the effect of a certain treatment. For each patient we measure the increase in a continuous response variable after treatment compared to before treatment. Let Y_{ij} be the measured increase for patient i in country j, for $i = 1, 2, \dots, n_j$. We assume a Bayesian hierarchical model in which, given the parameters, Y_{ij} has the normal distribution $\mathrm{N}(\alpha_j, \tau^{-1})$, with in turn $\alpha_j \sim \mathrm{N}(\mu, \omega^{-1})$, and priors on the parameters $\mu \sim \mathrm{N}(0, 1)$, $\tau \sim \mathrm{Gamma}(2, 1)$ and $\omega \sim \mathrm{Gamma}(1, 1)$.

   (a) justify the assumptions in terms of exchangeability

   (b) draw a DAG to represent the model, using 'plates' as in the figures on page 4 of lecture 5.

   (c) write out the joint distribution of all the variables

   (d) what is the distribution of α_1 given the observed data and all other parameters?

5. Look at the two undirected graphs G_1 and G_2, each with nodes labelled $\{1, 2, 3, \dots, 7\}$, and edges – for G_1: $\{1-2, 1-4, 1-5, 1-6, 1-7, 2-3, 2-6, 4-6, 4-7, 5-6, 6-7\}$ – and for G_2: $\{1-2, 1-5, 1-6, 1-7, 2-3, 2-6, 3-4, 4-5, 5-6, 6-7\}$.

   (a) write down the cliques for each graph

   (b) which of the two, if either, is decomposable?

   (c) find a junction tree for whichever is/are decomposable, checking the running intersection property (slide 20 of lecture 6).

6. For an ordinary Markov chain $(x_0, x_1, x_2, \dots, x_T)$, check that the moral graph is decomposable, and write down the junction tree.

7. Use Hugin to study the following problem (the 'coins' example may be useful as a partial template). A certain disease may take a harmless or a serious form, with probabilities in the population of 0.99 and 0.01 respectively. A blood test can be used to help decide, but it is error-prone. For a patient with the serious form of the disease, the test will indicate positive with probability 0.98, but it also indicates positive with probability 0.04 when the patient has only the harmless form of the disease. The test can be repeated, and the results are (conditionally) independent. Set up a Bayes Net with 3 nodes – severity of disease, number of tests (allow say up to 4), and number of positive results.

   What is the probability that a patient has the serious form of the disease, when he has 1 positive result in 1 test? 2 positives in 2 tests? 2 positives in 4 tests?