MATH11400 Statistics 1 2010–11

Homepage http://www.stats.bris.ac.uk/%7Emapjg/Teach/Stats1/

Hypothesis Tests, continued

8.7 Example of 1-sample t-test

To investigate the accuracy of D-I-Y radon detectors, researchers bought 12 such detectors and exposed them to exactly 105 picocuries per litre of radon. The 12 detector readings were:
91.9, 97.8, 111.4, 122.3, 105.4, 95.0, 103.8, 99.6, 96.6, 119.3, 104.8, 101.7.

This gives summary statistics $\sum_i x_i = 1249.7$, $\overline{x} = 104.1417$, $n = 12$, $\sum_i x_i^2 = 131096.44$, $S^2 = 86.4181$.

Our question: does the mean for such detectors seem to differ from 105?

1. Model assumptions: these 12 observations are a random sample from $N(\mu, \sigma^2)$ where both μ and σ^2 are unknown.

2. Hypotheses: $H_0 : \mu = 105$ vs. $H_1 : \mu \neq 105$ (i.e. 2-sided alternative).

3. Test statistic:

$$T = \frac{\overline{X} - \mu_0}{S/\sqrt{n}} \sim t_{n-1} \quad \text{when } H_0 \text{ is true}$$

We have $n = 12$ and $\mu_0 = 105$, so the observed value of T is $t_{obs} = \sqrt{12}(104.142 - 105)/\sqrt{86.4181} = -0.32$ and $|t_{obs}| = 0.32$.

4a. p-value approach. The p-value is $P(|T| > 0.32)$ when $T \sim t_{n-1} = t_{11}$. Using **R**, we find `pt(0.32,11)=0.6225` so $P(|T| > 0.32) = 2(1 - 0.6225) = 0.755$. This is not a small probability!

4b. Critical region approach. For an α level test we use the critical region $C = \{|T| \geq t_{11;\alpha/2}\}$. Let us take the significance level $\alpha = 0.05$. Then $t_{11;0.025} =$ `qt(0.975,11)=2.201`. The value of t_{obs} is not within $C = \{|T| \geq 2.201\}$, so there is no reason to reject H_0 at the 5% level.

Conclusion. Taking either the p-value or critical region approach, there is no reason to reject the null hypothesis that the detectors are accurate.