I. THEORETICAL ANALYSIS

We consider water wave propagation in a main guide of width $d$ supporting two contiguous channels of width $2a$ and heights $(b_1, b_2)$. Using the assumptions of an inviscid, incompressible fluid, and irrotational motion, the linearized equation for the velocity potential $\phi(x, y, z)$ is

$$\Delta \phi = 0, \quad \frac{\partial \phi}{\partial z}(x, y, 0) = \frac{\omega^2}{g} \phi(x, y, 0), \quad \nabla \phi \cdot \mathbf{n}_\Gamma = 0,$$

with $g = 9.81 \text{ m.s}^{-2}$ the gravity and $\omega$ the frequency (in the harmonic regime with time dependence $e^{-i\omega t}$). $z$ is the vertical coordinate with $z = 0$ the undisturbed free surface and $z = -b$ the sea bottom ($x$ the axis of the guide); $\Gamma$ denotes the boundaries of the vertical walls of the guide and side channels and the horizontal (rigid) sea bottom. The free surface elevation $\eta(x, y)$ is defined as

$$\eta(x, y) = \frac{i\omega}{g} \phi(x, y, 0).$$

With $h$ the constant water depth, the above equations simplify to the two-dimensional Helmholtz equation

$$(\nabla^2 + k^2)\eta = 0, \quad \nabla \eta \cdot \mathbf{n}_\gamma = 0, \quad (1)$$

where $\gamma$ denotes the rigid boundaries of the guide and side channels reduced to segments in the $(x, y)$ plane and where $k$ is the wavenumber satisfying the dispersion relation $\omega^2 = gk \tan(kb)$. In the main waveguide of width $d$, we shall only need that the solution can be approximated by $\eta(x, y) \simeq \eta(x)$ governed by $\eta''(x) + k^2 \eta(x) = 0$. Similarly, in the two channels, we have $\eta(x, y) \approx \eta_n(y)$, $n = 1, 2$, governed by $\eta''_n(y) + k^2 \eta_n(y) = 0$. With $\eta_s = \eta_1 + \eta_2$, $\eta_s = \eta_1 - \eta_2$, being the symmetric and antisymmetric fields in the side channels and we define $\bar{\eta} = \frac{1}{2}(\eta^s + \eta^a)$ and $|\eta| = (\eta^s - \eta^-)$ to be the average and the jump of $\eta$ across the junction at $x = 0$ ($\eta^s = \eta(0^+)$.)

$$[\eta] = 2\pi d |\eta| + d |\eta'| + a_n(0), \quad d[\eta'] + a_n(0) = 0,$$

$$\eta_s(0) = 2\eta + \frac{B}{2} a_n(0), \quad \eta_s(0) = -\frac{B}{2} a_n(0). \quad (2)$$

In [? ], the above jump conditions have been demonstrated and the analysis has been further developed for identical side channels. Here we extend the result to side channels of different heights $b_1 = b(1 - \varepsilon)$ and $b_2 = b(1 + \varepsilon)$. The solution for an incident wave $e^{ikx}$ reads

$$\eta(x) = e^{ikx} + R e^{-ikx}, \quad x \in (-\infty, 0), \quad \eta_n(y) = A_n \cos(k(y - b_n)), \quad n = 1, 2, \quad \eta(x) = T e^{ikx}, \quad x \in (0, +\infty).$$

The scattering coefficients $(R, T)$ are deduced from (2) and they read

$$T = \frac{\Re(z_n^* - z^2)}{z_n^* z_n^* + z^2}, \quad R = -\frac{i\Im(z_n z^*) + 2z}{z_n^* z_n^* + z^2}. \quad (3)$$

where, introducing $x_n = ka \tan(kb_n), n = 1, 2$,

$$z_n = 1 + \frac{i\gamma_n}{2kd}, \quad z_n = 1 + \frac{k\gamma_n}{2}, \quad z = -\frac{B}{2D} (x_1 - x_2),$$

$$\gamma_n = B + \frac{B^2}{2D} (x_1 + x_2 - 2B x_1 x_2), \quad \gamma_n = -\frac{2}{D} (x_1 + x_2 - 2B x_1 x_2),$$

with $D = 1 - \frac{1}{2}(B_s + B_s)(x_1 + x_2) + B_s B_s x_1 x_2$. Note that the parameters used in the main document are linked to those defined in [? ] by the relations

$$\delta_0 = \frac{\pi}{4} B_s, \quad \delta_0 = \frac{\pi}{4} B_s, \quad \delta_0 = \frac{\pi}{8} (B_s + B_s - B), \quad \delta = \frac{\pi}{4} B_s,$$

which allow for more compact expressions. In the main document, Eqs. (8-9) correspond to Taylor expansions of $(R, T)$ in (3) near the resonance at $kb = \pi/2$ (and (4-5) to the case of identical channels with $b_1 = b_2$ as in [? ]).

An important result given by this analysis is that two separate transmission zeros are obtained and, depending on the sign of

$$\Delta = \Delta_0 + \left(\frac{\pi b}{2a}\right)^2 \varepsilon^2, \quad \Delta_0 = (\delta_0 - \delta_0) - \delta^2, \quad (4)$$

they correspond to real ($\Delta > 0$) or complex conjugate ($\Delta < 0$) wavenumbers $k$. We also stress that in (4) $\Delta_0$ depends only on the geometry of the junction as the dimensionless parameters $(\delta_0, \delta_0, \delta_0, \delta)$ do, and the dependence of $\Delta$ on $\varepsilon = (b_2 - b_1)/(b_1 + b_2)$ is explicit.
II. DERIVATION OF THE RELATION BETWEEN $A_a$ AND $A_s$

The analysis of the experimental results in figure 6 (main text) suggests that PA is achieved for a particular balance between the symmetric $A_s = (A_1 + A_2)$ and antisymmetric $A_s = (A_1 - A_2)$ complex amplitudes in the channels, of the form $A_s = i \tan(ka/2)A_a$. Below we show that this relationship is consistent with a calculation in which the two channels are replaced by two point sources separated by a distance $a/2$ (at the center bottom of each channel) and imposing corresponding amplitudes $A_1$ and $A_2$. To do so, we introduce the Green’s function $g(x, y)$ satisfying

$$(\nabla^2 + k^2)g(x, y) = 0, \quad x \in (-\infty, +\infty), \quad y \in (0, d),$$

along with the boundary conditions $\frac{\partial g}{\partial y}(x, 0) = 0$, and $\frac{\partial g}{\partial y}(x, d) = \delta(x)$. We then have

$$\lim_{|x| \to +\infty} g(x, y) = \frac{ie^{ik|x|}}{2kd},$$

see e.g. [?]. We now consider the field $\varphi(x, y)$ generated in the waveguide by two point sources on the upper wall at $y = d$, one located at $x = -a/2$ of amplitude $A_1$ and one located at $x = a/2$ of amplitude $A_2$. From above we have

$$\lim_{|x| \to +\infty} \varphi(x, y) = \frac{i}{2kd} \left( A_1 e^{ik|x+a/2|} + A_2 e^{ik|x-a/2|} \right).$$

Highly non-symmetric emission by these two sources is obtained, for what we are interested in, when the radiated field vanishes at $x \to -\infty$, which is obtained when $A_1 = -A_2 e^{ika}$. We notice that this is equivalent to superposing a symmetric source with amplitude $A_s = (A_1 + A_2)$ and an anti-symmetric source with amplitude $A_a = (A_1 - A_2)$ satisfying

$$A_s = i \tan(ka/2)A_a,$$

as observed experimentally for which we have $\tan(ka/2) = 0.48$ at the PA.