Trapping and near-trapping of waves by a circular island

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Under the assumptions of shallow water theory, it is shown that a class of axisymmetric bathymetry surrounding a circular island can trap waves perfectly. However, these exact solutions require the fluid depth to increase indefinitely with distance from the island, violating the assumptions of the shallow water model. The depth-averaged mild-slope equations extend the shallow water theory to fluid of all depths and are used to demonstrate that the trapped waves predicted by shallow water theory are manifested by large amplitude resonances (a signature of near trapping) in a wave scattering problem.

1. Introduction

The refraction of surface gravity waves by changes in the fluid depth occurs as a consequence of the dispersion relation which connects the phase speed of waves to the fluid depth. This allows a phenomenon analogous to total internal reflection to occur as oblique waves propagate from shallow water into deeper water. This principle can be used to argue that waves can be trapped by a straight coastline or shoreline away from which the fluid depth increases. These so-called edge waves are characterised by a longshore wavelength with attenuation away from the shoreline. The most notable example is Stokes's (1847) simple closed form solution for edge waves along a plane sloping beach which exist for all angles of beach. A sequence of further edge wave solutions emerge as the beach angle is reduced (see Ursell (1952)). The fluid depth does not need to tend to infinity for edge wave solutions to exist. For example, waves can be trapped above infinitely-long rectangular shelves (Evans & McIver (1984)) protruding from an otherwise constant finite depth bed. Waves can also be trapped above long submerged obstacles which are not connected to the bed, such as long submerged cylinders (e.g. Ursell (1951).)

Edge waves are of practical interest as it has been suggested that they are responsible for the longshore transport of sand and the formation of cusp-like structures on beaches (Minzoni & Whitham (1977)). Other geophysical effects have been added to the basic fluid model, such as stratification and rotation (see Llewellyn Smith (2004).)

Extensions of the edge wave solutions have also been sought when the shoreline is no longer linear but circular. Longuet-Higgins (1967) demonstrated, using shallow water theory, that it is not possible to trap waves on an island when the fluid depth is assumed to tend to a constant at infinity. Nevertheless, in his work, near trapped waves were identified and it was demonstrated by the resonant amplification effects on incoming wave fields. Subsequently, work based on full linear theory has also identified large resonant effects due to submerged circular sea mounts and islands supported by variable bathymetry. See for example, Renardy, Chamberlain etc etc.

In this paper we initially return to the formulation of Longuet-Higgins (1967), and the use of shallow water theory. We relax the insistence that the depth must tend to a constant value at infinity and allow it to increase without bound – in spite of the clear violation of the use of shallow water theory. As argued by Longuet-Higgins (1967) it is shown that trapped waves are possible for bed shapes with depths that grow algebraically faster than r^2 where r is the distance from the centre of the island. Here we produce the solutions in closed form. The justification for presenting these solutions is that the decay of the trapped wave solution away from the island is relatively fast and hence the solution is invalidated by a region which may have relatively little effect. Thus these solutions, although approximate, may indicate the presence of trapped wave solutions for a similar type of bathymetry when formulated under full linear theory.

In Section 2 we formulate the shallow water model and present explicit trapped wave solutions in terms of Bessel functions for a class of bathymetry with depth given by a power of the radius. Certain solutions are proved to exist, others not. In Section 3 the effect of these shallow water trapped waves are considered in another approximate model which extends shallow water theory. The modified mild-slope equations (see Porter (2003)), still have the advantage of belonging to the class of depth-averaged models, but place no restriction on the fluid depth. The approximation comes about from assuming that the gradient of the bed with respect to the product of the local wavenumber and the depth is small. In this case we consider a scattering problem in which plane waves propagate from infinity over fluid of constant depth and are incident upon a finite circular domain containing the axisymmetric bathmetry and island predicted by shallow water theory. The aim here is to establish the effect of the influence of exact trapped modes predicted by shallow water theory upon a more sophisticated wave propagation model.

2. Shallow water theory and trapped waves

A vertical-sided cylindrical island of radius a is centred at the origin. It is surrounded by water of depth $h(r)$ which varies only as a function of the radius r.

Under depth-averaged shallow water theory for a bed of variable depth h the governing equation for the surface elevation $\eta(r,\theta)$ having had an assumed time harmonic dependence of angular frequency ω removed, is given by

$$\nabla \cdot (h\nabla\eta) + \frac{\omega^2}{g}\eta = 0, \qquad r > a. \tag{2.1}$$

and satisfies an impermeable condition on the cylinder

$$\eta_r(a,\theta) = 0. \tag{2.2}$$

Since we seek a trapped wave localised around a circular island and it is certainly necessary that

$$\eta(r,\theta) \to 0, \qquad \text{as } r \to \infty. \tag{2.3}$$

More specifically, the trapped wave must possess finite energy, E, which, in this depth averaged context, is defined as

$$E = \frac{1}{2}\iint_D \left(h(\nabla\eta)^2 + \frac{\omega^2}{g}\eta^2 \right) r dr d\theta \tag{2.4}$$

and D represents the two-dimensional domain $r > a$, $0 \le \theta < 2\pi$. We remark that the energy functional, written as the sum of kinetic and potential energy components, $E = T + V$ can be used to derive the governing equation (2.1) and boundary condition (2.2) by setting $\delta(T - V) = 0$ by Hamilton's principle.

Since the bathmetry is axisymmetric, we seek a solution of the form $\eta(r,\theta) = \zeta_m(r)\cos m\theta$, $m = 0, 1, \ldots$ which reduces (2.1) to

$$(r\,h(r)\zeta_m'(r))' + (\omega^2/g - m^2 h(r)/r^2) r\zeta_m(r) = 0, \qquad r > a \tag{2.5}$$

with (2.2) giving $\zeta_m'(a) = 0$.

Introducing dimensionless variables,

$$s = r/a, \qquad h(r) = h(a)\sigma(s), \qquad \lambda^2 = \frac{\omega^2 a^2}{gh(a)} \tag{2.6}$$

which requires us to set $\sigma(1) = 1$, and letting $\zeta_m(r) = y_m(s)$ gives

$$(s\sigma(s)y_m'(s))' + (\lambda^2 s - m^2\sigma(s)/s)y_m(s) = 0, \qquad s > 1 \tag{2.7}$$

with $y_m'(1) = 0$. In terms of transformed variables a trapped wave also requires E_m to be finite where

$$E_m = \frac{\pi h(a)}{2}\int_1^\infty \left(\sigma(s)(y_m'(s))^2 + (\lambda^2 + m^2\sigma(s)/s^2)(y_m(s))^2\right) s ds \tag{2.8}$$

and the angular dependence has been integrated explicitly. A necessary, but not sufficient, condition for E_m to be finite is that $y_m(s) \sim s^{-p}$ as $s \to \infty$ where $p > 1$.

For simplicity, we consider bed shapes given by $\sigma(s) = s^{2\beta}$. There is a critical value of $\beta = 1$ where the character of the solutions change as we shall show.

When $\beta = 1$ (the bed is quadratic), general solutions of (2.7) are given by

$$y_m(s) = As^{-1-\sqrt{1+m^2-\lambda^2}} + Bs^{-1+\sqrt{1+m^2-\lambda^2}} \tag{2.9}$$

In order to satisfy the boundedness condition at infinity we require $\lambda^2 < 1 + m^2$ and $B = 0$, but we cannot subequently satisfy $y_m'(1) = 0$. Consequently, trapped waves are not possible for $\beta = 1$.

For $\beta < 1$ general solutions of (2.7) are given by

$$y_m(s) = As^{-\beta}J_{-\nu}\left(\frac{\lambda s^{1-\beta}}{1-\beta}\right) + Bs^{-\beta}Y_{-\nu}\left(\frac{\lambda s^{1-\beta}}{1-\beta}\right), \qquad \text{with } \nu = \frac{\sqrt{\beta^2+m^2}}{(\beta-1)} \tag{2.10}$$

and as $s \to \infty$

$$s^{-\beta}J_{-\nu}\left(\frac{\lambda s^{1-\beta}}{1-\beta}\right) = O(s^{-(1+\beta)/2}), \qquad s^{-\beta}Y_{-\nu}\left(\frac{\lambda s^{1-\beta}}{1-\beta}\right) = O(s^{-(1+\beta)/2}), \tag{2.11}$$

(e.g. §9.2.1, §9.2.2 in Abramowitz & Stegun). Neither of these components of $y_m(s)$ decay fast enough as $s \to \infty$ to satisfy the energy criterion for trapped waves.

Finally, we turn to $\beta > 1$. Now general solutions of (2.7) are expressible as

$$y_m(s) = As^{-\beta}J_\nu\left(\frac{\lambda s^{1-\beta}}{\beta-1}\right) + Bs^{-\beta}Y_\nu\left(\frac{\lambda s^{1-\beta}}{\beta-1}\right), \tag{2.12}$$

with ν as in (2.10). As $s \to \infty$ the asymptotics of the two functions (e.g. §9.1.7, §9.1.9 of Abramowitz & Stegun) are

$$s^{-\beta}J_\nu\left(\frac{\lambda s^{1-\beta}}{\beta-1}\right) = O(s^{-\beta-\sqrt{\beta^2+m^2}}), \qquad s^{-\beta}Y_\nu\left(\frac{\lambda s^{1-\beta}}{\beta-1}\right) = O(s^{-\beta+\sqrt{\beta^2+m^2}}). \tag{2.13}$$

Thus, the second function must be excluded from the general solution as it does not decay as $s \to \infty$ (i.e. $B = 0$). But the first solution decays at least as fast as $O(1/s^{2\beta})$ and this is always enough to make E_m finite for all $\beta > 1$.

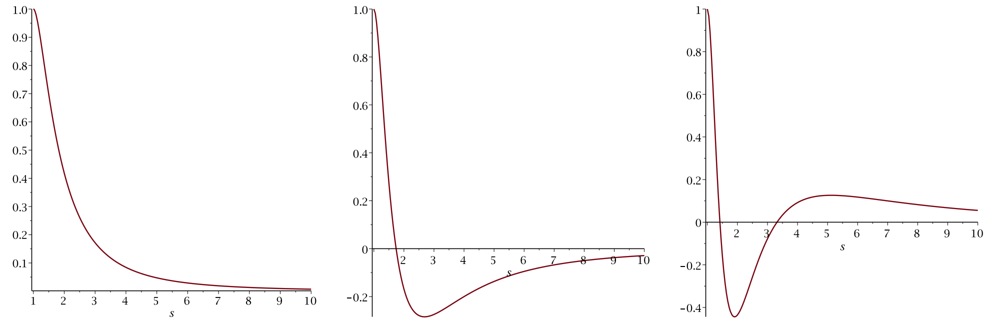

FIGURE 1. Trapped wave solutions $y_0(s)$ for $\beta = 3/2$.

This leaves us with the solution

$$y_m(s) = s^{-\beta} J_\nu \left(\frac{\lambda s^{1-\beta}}{\beta - 1} \right) \Big/ J_\nu \left(\frac{\lambda}{\beta - 1} \right) \tag{2.14}$$

and the arbitrary constant A has been chosen to normalise the solution such that $y_m(1) = 1$. Application of the condition $y_m'(1) = 0$ furnishes the following condition to be satisfied by λ for trapped wave solutions to exist:

$$\lambda J_{\nu+1} \left(\frac{\lambda}{\beta - 1} \right) = (\beta + \sqrt{\beta^2 + m^2}) J_\nu \left(\frac{\lambda}{\beta - 1} \right). \tag{2.15}$$

We note that the character of the Bessel functions changes from algebraic to oscillatory as the argument increases beyond the the order (e.g. §9.3.35 Abramowitz & Stegun) in a manner related to the transition in character of the Airy function across the origin. Thus we expect solutions of (2.15) when

$$\lambda \gtrsim \sqrt{\beta^2 + m^2} \tag{2.16}$$

and the oscillatory nature of the Bessel functions means that we should expect an infinite sequence of solutions with increasing λ.

2.1. *Example: A bed with a cubic depth profile* ($\beta = \frac{3}{2}$)

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With $\beta = \frac{3}{2}$ and $m = 0$, (2.15) is $\lambda J_4(2\lambda) = 3J_3(2\lambda)$ which reduces to

$$J_1(2\lambda) = \lambda J_0(2\lambda) \tag{2.17}$$

and this is satisfied by $\lambda = 2.567811, 4.208622, 5.809920$ etc with values tending asymptotically to $\lambda = \frac{1}{2}p\pi - \frac{1}{8}\pi$, p in integer, as $p \to \infty$.

Under this condition, the wave elevation is inferred from the function

$$y_0(s) = s^{-3/2} J_3 \left(\frac{2\lambda}{\sqrt{s}} \right) \Big/ J_3 (2\lambda) \tag{2.18}$$

which decays like $1/s^3$ as $s \to \infty$. The first 3 modes are shown in Fig. 1.

For $m = 1$, the solution is in terms of Bessel functions of fractional order, so λ is given by

$$\lambda J_{\sqrt{13}+1}(2\lambda) = \tfrac{1}{2}(3 + \sqrt{13}) J_{\sqrt{13}}(2\lambda) \tag{2.19}$$

and we find the sequence $\lambda = 2.891960488, 4.584528953, 6.210306175$ etc, whilst the wave

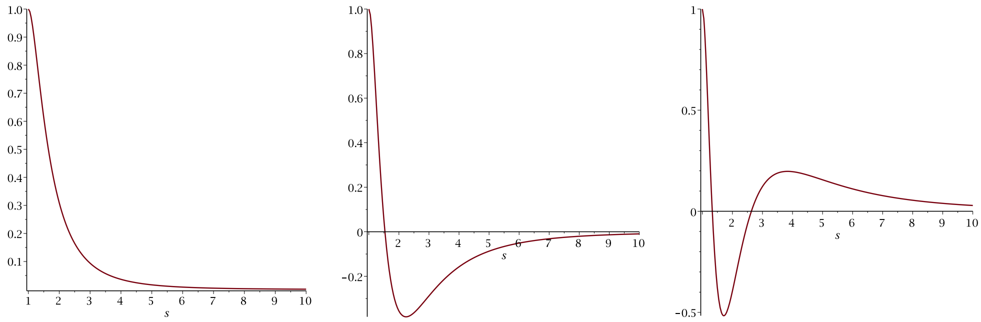

FIGURE 2. Trapped wave solutions $y_2(s)$ for $\beta = 3/2$.

elevation is

$$y_1(s) = s^{-3/2} J_{\sqrt{13}}\left(\frac{2\lambda}{\sqrt{s}}\right) \Big/ J_{\sqrt{13}}(2\lambda) \tag{2.20}$$

For $m = 2$, (2.15) reduces to the condition

$$\lambda J_4(2\lambda) = J_5(2\lambda) \tag{2.21}$$

which has a sequence of roots, $\lambda = 3.632342098$, 5.433869608, 7.113189027 etc.

Now the solution is

$$y_2(s) = s^{-3/2} J_5\left(\frac{2\lambda}{\sqrt{s}}\right) \Big/ J_5(2\lambda) \tag{2.22}$$

and this decays like $1/s^4$ as $s \to \infty$. The first 3 modes are shown in Fig. 2

Fundamental modes for $m = 3, 4, 5$ are given by $\lambda = 4.531654241$, 5.492021591, 6.479411101.

There is a problem with these results: shallow water theory requires the water depth to be shallow and the solutions shown here for for depths increasing indefinitely to infinity. The next section explores if the existence of these shallow water trapped waves solutions are felt by a more widely applicable theory of wave propagation, with the depth restriction removed.

3. Mild-slope equations and near-trapping of waves

We now consider a scattering problem in which plane monochromatic waves are incident from infinity over a flat bed of depth h_0 upon an axisymmetric shoaling region in $a < r < b$ of prescribed depth $h(r)$ (with $h(b) = h_0$) which meets the vertical-sided circular island at $r = a$.

In order to retain the simplicity of a depth-averaged model of wave propagation over a variable bed without the restriction of shallow water, we use Porter's (2003) version of the modified mild-slope equation (MMSE). This will not be as accurate as unapproximated full linear theory and includes the mild-slope restriction, $|h'/kh| \ll 1$ and k is the local wavenumber defined in (3.2) below. However, it can be regarded a natural extension of shallow water theory to water of any depth.

In the derivation of MMSE approximation, the total velocity potential describing the fluid flow, $\Phi(r, \theta, z)$, satisfying Laplace's equation and the linearised free surface condition, is approximated, in a variational principle, by assuming the separable form

$$\Phi(r, \theta, z) \approx \phi(r, \theta) \cosh k(z + h) \tag{3.1}$$

where $k = k(h)$ varies locally with the depth according to the usual dispersion relation

$$\frac{\omega^2}{g} = k \tanh kh. \tag{3.2}$$

When $r > b$ and the bed is flat with $h = h_0$, we write $k(h_0) = k_0$, and here the general solution is written

$$\phi(r, \theta) = \phi_i(r, \theta) + \phi_s(r, \theta) \tag{3.3}$$

where the incident wave is specified as

$$\phi_i(r, \theta) = \mathrm{e}^{\mathrm{i}k_0 x} = \sum_{m=0}^{\infty} \varepsilon_m \mathrm{i}^m J_m(k_0 r) \cos m\theta \tag{3.4}$$

($\varepsilon_0 = 1$, $\varepsilon_m = 2$ for $m \geq 1$) and the scattered waves are given by

$$\phi_s(r, \theta) = \sum_{m=0}^{\infty} \varepsilon_n \mathrm{i}^m A_m H_m(k_0 r) \cos m\theta. \tag{3.5}$$

Here $H_m(\cdot)$ is the Hankel function of the first kind, ensuring outgoing waves at infinity and A_m are undetermined coefficients (scattering amplitudes).

On account of the axisymmetric bathymetry and the series expansion of the solution in polar coordinates in $r > b$, we write

$$\phi(r, \theta) = \sum_{m=0}^{\infty} \epsilon_m \mathrm{i}^m \phi_m(r) \cos m\theta \tag{3.6}$$

for all $r > a$. Thus, for $r > b$, where the bed is flat,

$$\phi_m(r) = J_m(k_0 r) + A_m H_m(k_0 r). \tag{3.7}$$

Over $a < r < b$ the solution is determined using the MMSE of Porter (2003, equation (2.4)) which reduces here to the equation

$$(rk^{-2}\zeta_m'(r))' + r(1 - v(h)h'^2 - (m/kr)^2)\zeta_m(r) = 0 \tag{3.8}$$

where

$$v = (3(2K + \sinh K)(\sinh(2K) - \sinh K) - 3K^2(\cosh(2K) + 2) - 4K^3 \sinh K - K^4) \\ /(3(K + \sinh K)^4) \tag{3.9}$$

with $K = 2kh$, an equation which is finally arrived at via a rescaling of the dependent variable:

$$\phi_m(r) = \zeta_m(r) \frac{k(h)}{k_0} \sqrt{\frac{u(h)}{u(h_0)}} \tag{3.10}$$

in which

$$u(h) = \frac{(2kh + \sinh(2kh))}{4k \cosh^2(kh)}. \tag{3.11}$$

The normalisation in (3.10) is chosen such that values of $\phi_m(r)$ and $\zeta_m(r)$ coincide on $r = b$. The solution to (3.8) in $a < r < b$ must match the solution (3.7) on the boundary $r = b$ and satisfy the no-flow condition, $\zeta_m'(a) = 0$ (equivalent to $\phi_m'(a) = 0$) on the other boundary.

We use the same non-dimensionalisation used in the previous section, namely $s = r/a$,

$h(r) = h(a)\sigma(s)$ as well as writing

$$y_m(s) = \zeta_m(r), \qquad z_m(s) = rk^{-2}\zeta_m'(r). \tag{3.12}$$

Then (3.8) can be written as the coupled 1st order system

$$\mathbf{y}_m'(s) = \mathsf{A}_m(s)\mathbf{y}_m(s), \tag{3.13}$$

where

$$\mathbf{y}_m(s) = \begin{pmatrix} y_m(s) \\ z_m(s) \end{pmatrix}, \quad \mathsf{A}_m(s) = \begin{pmatrix} 0 & \kappa^2(s)/s \\ s(1 - w(s)\sigma'^2(s) - m^2/(\kappa(s)s)^2) & 0 \end{pmatrix} \tag{3.14}$$

for $1 < s < \hat{b}$ where $\hat{b} = b/a$, $w(s) = v(h)\hat{h}^2$, $\hat{h} = h(a)/a$ and $\kappa(s) = k(h)a$ so that (3.2) becomes

$$\hat{h}\lambda^2 = \kappa(s)\tanh(\hat{h}\kappa(s)\sigma(s)) \tag{3.15}$$

with $\lambda^2 = \omega^2 a/(g\hat{h})$ coinciding with the definition in §2. Then $\kappa_0 \equiv k_0 a$ is the root of

$$\hat{h}\lambda^2 = \kappa_0 \tanh(\hat{h}\kappa_0\sigma(\hat{b})). \tag{3.16}$$

We let $\mathbf{y}_m(s) = (1+A_m)\mathbf{y}_m^{(1)}(s) + \mathrm{i}A_m\mathbf{y}_m^{(2)}(s)$ where $\mathbf{y}_m^{(i)}(s)$ for $i = 1, 2$ both satisfy (3.13) with

$$\mathbf{y}_m^{(1)}(\hat{b}) = \begin{pmatrix} J_m(\kappa_0\hat{b}) \\ (\hat{b}/\kappa_0)J_m'(\kappa_0\hat{b}) \end{pmatrix}, \qquad \mathbf{y}_m^{(2)}(\hat{b}) = \begin{pmatrix} Y_m(\kappa_0\hat{b}) \\ (\hat{b}/\kappa_0)Y_m'(\kappa_0\hat{b}) \end{pmatrix}. \tag{3.17}$$

We numerically integrate solutions $\mathbf{y}_m^{(i)}$ governed by (3.13) from $s = \hat{b}$ to $s = 1$ with the initial conditions (3.17) and can therefore determine the pair of unknowns A_m and $y_m(1)$ from the application of the homogeneous Neumann condition at $s = 1$:

$$(1 + A_m)\mathbf{y}_m^{(1)}(1) + \mathrm{i}A_m\mathbf{y}_m^{(2)}(1) = \begin{pmatrix} y_m(1) \\ 0 \end{pmatrix}. \tag{3.18}$$

3.1. *Results*

The amplitude of waves on the boundary of the island, $r = a$, relative to that of incoming waves, is given by

$$\phi(a, \theta) = \frac{k(h(a))}{k_0}\sqrt{\frac{u(h(a))}{u(h_0)}}\sum_{m=0}^{\infty} \varepsilon_m \mathrm{i}^m y_m(1)\cos m\theta \tag{3.19}$$

The scaling factor premultiplying the series in (3.19) encodes wave amplification due to shoaling. This can be regarded as a background effect which does nothing to indicate the presence of a resonance around the island. It is therefore the coefficients $y_m(1)$ which indicate additional effects. Since $y_m(1)$ scales linearly with the modal amplitudes A_m, we plot $|A_m|$, $m = 0, \ldots, 4$ against wave frequency in the results presented here to indicate the presence and strength of resonance or near-trapping.

We consider first the effect of the bed shape β on the scattering amplitudes. In figures 3(a,b,c) we set $\beta = 0.95$, $\beta = 1$ and $\beta = 1.05$, respectively just below, at and above the critical index required by shallow water theory for trapped waves. We have fixed the shoaling ratio $h(a)/h(b) = 1/125$ and set the size of the island such that $\hat{h} \equiv h(a)/a = 1/100$. Thus in figures 3(a,b,c) we show the frequency parameter λ on the horizontal axis and $|A_m|$ on the vertical axis (all the same scale) for $m = 0, 1, 2, 3, 4$ (the curves are ordered left to right respectively in each figure).

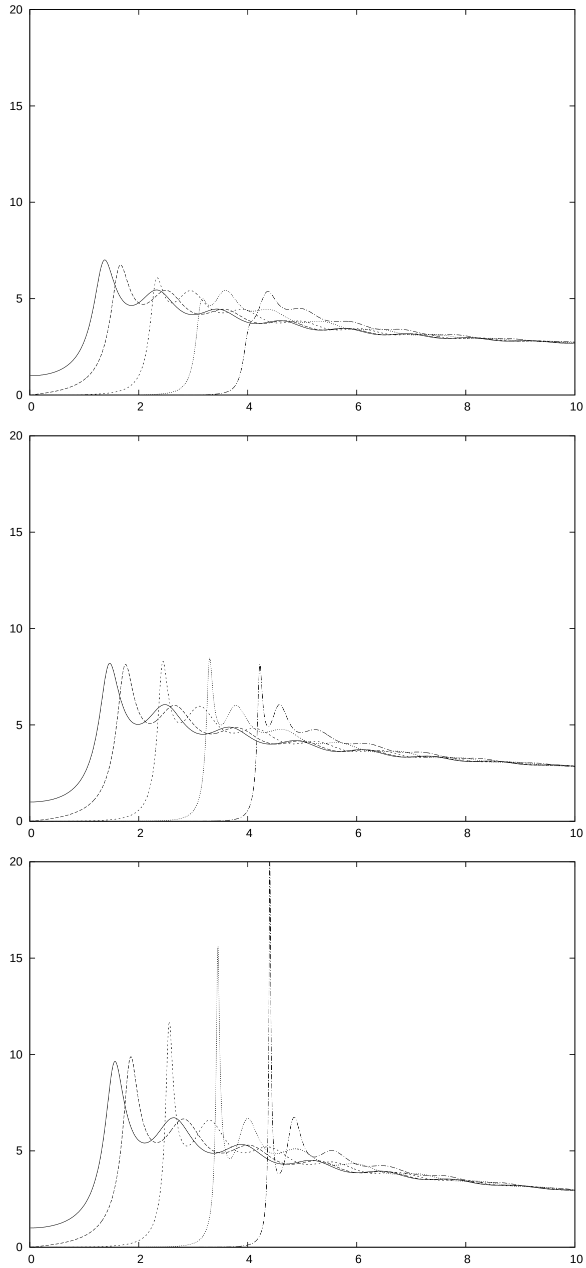

FIGURE 3. The amplitude of the MSE scattering coefficients $|A_m|$ against frequency parameter λ for beds with $\beta = 0.95$, 1 and 1.05. In each figure, $h(a)/h(b) = 1/125$ and $h(a)/a = 1/100$.

We observe a clear transition in the behaviour of these curves as β passes through $\beta = 1$.

We focus now on the particular case $\beta = 3/2$. In figure 4(a) we have set $\hat{b} \equiv b/a = 5$ (this implies $h(a)/h(b) = 1/125$) and $\hat{h} \equiv h(a)/a = 1/100$. The fundamental frequencies for perfectly trapped modes predicted by shallow water theory are superimposed on the horizontal axis. There is a clear correspondance between the sharp peaks in the scattering coefficients and the frequencies at which trapped modes occur according to shallow water theory. These peaks increase in sharpness as m increases. The signature of higher order modes is much reduced with only a small peak for the second predicted mode with $m = 4$ for example. In figure 4(b) we double the size of the shoaling domain to $\hat{b} = 10$ keeping $\hat{h} = 1/100$ as before. The peaks in the scattering amplitudes associated with fundamental modes are increased in sharpness and the second order resonances also show signs of increasing in sharpness. Finally, in figure 4(c) we show the effect of reducing $\hat{h}$ from 1/100 to 1/10 so that the water is not as shallow. Here, the signature of trapped modes from shallow water theory is greatly reduced.

We can make sense of these results by observing how rapidly the perfectly trapped modes under shallow water theory decay in figures 1, 2 as a function of s. Thus, the fundamental modes decay more rapidly than higher order modes, whilst the rate of decay also increases with increasing m. In terms of exciting these modes over a finite domain, it is evident that those modes which decay fastest will be excited more.

4. Conclusion

We have shown that under an approximate shallow water theory, waves can be perfectly trapped to a circular island on axisymmetric bathymetry such that the water depth grows at a faster rate than quadratic with the distance from the island. These modes, which are deduced explicitly, and the corresponding sets of eigenfrequencies appear of limited significance as solutions violate the model assumptions.

However, results from an approximation leading to the Mild-Slope Equation and, valid for all water depths, indicate a strong correlation between the existence of the shallow water trapped modes and resonances or near-trapping in the scattering by a finite circular shoaling region. The shallowness of the water does play an important role and if $h(a)/a$ is taken to be too large (here we have shown values of 1/10) then the signature of the trapped modes is lost.

The identification of the trapped modes described in Section 2 is consistent with classic Sturm-Liouville theory, although no use of this theory has been made here.

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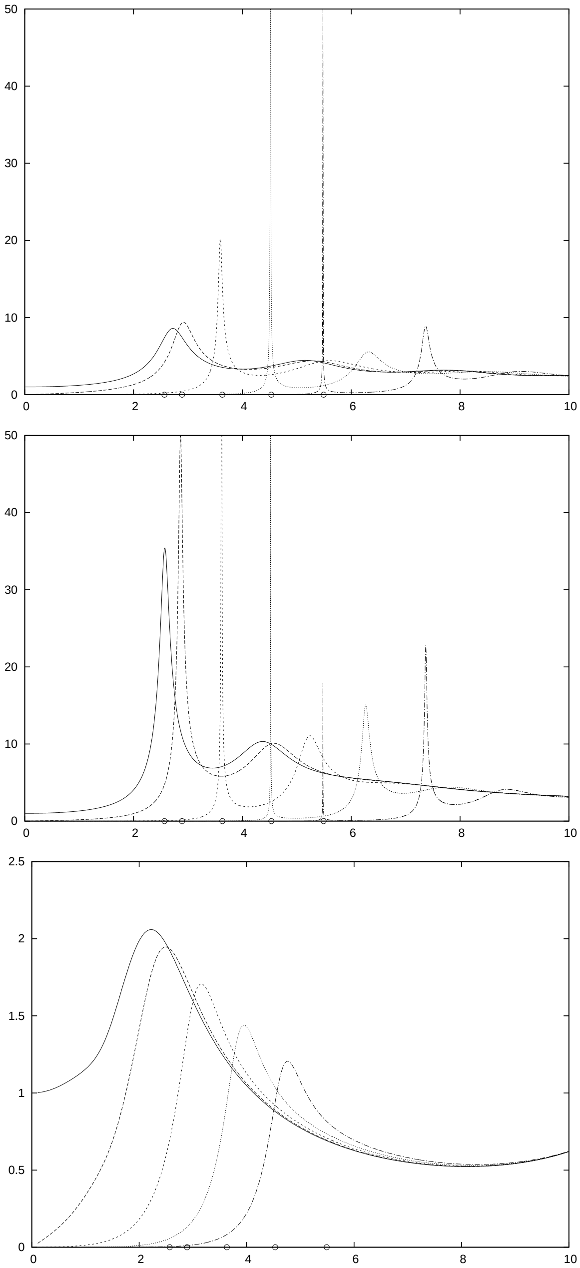

FIGURE 4. The amplitude of the MSE scattering coefficients $|A_m|$ against frequency parameter λ for a cubic bed, $\beta = \frac{3}{2}$. In (a) $h(a)/h(b) = 1/125$ and $h(a)/a = 1/100$; in (b) $h(a)/h(b) = 1/1000$ and $h(a)/a = 1/100$; in (c) $h(a)/h(b) = 1/125$ and $h(a)/a = 1/10$.