Strongly nonlinear evaluation of internal ship wakes

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- ship internal wave wakes at supercritical speed;
- strongly nonlinear interfacial model accounting for realistic ship geometries of draught comparable to the average depth of the pycnocline;
- similarities and differences between two- and three-layer fluids;
- numerical wake-induced amplitudes, currents, strain rate, with comparison to field experiments.

Background. There is a recent interest in internal wave wakes, generated by a ship moving in a stratified sea, at supercritical speed (Daniel Bourgault, 2014, personal communication). Watson et al. (1992) gives a summary of this research up to that date. Recent analyses and calculations given in the IWWWFB-community are obtained applying pressure distributions (Parau et al., 2007). Watson et al. (1992) analysed aspects of the ship generated internal waves using data from a set of experiments, with three different ships, in Loch Linnie, Scotland in 1989. They presented results for the wave wake amplitudes, wake-induced currents and quantity such as the strain rate at the sea surface. While existing models, at that time, basically assumed ships of very small draught, Watson et al. requested prediction tools that could allow for a finite draught of the ship. Such calculations are performed in the present account, with realistic ship models of draught comparable to the average depth of the pycnocline (see figure 3a,b).

Nonlinear interfacial model. Let $\mathbf{x} = (x_1, x_2)$ denote horizontal coordinates and y be vertical. $y = 0$ coincides with the interface at rest. A two-layer fluid has constant densities ρ_0 and $\rho_1 = \rho_0 + \Delta\rho$, where index 0 refers to the upper layer and index 1 to the lower. Layer depths at rest are h_0 and h_1, respectively. Assuming incompressible and irrotational motion in each of the layers, the fluid motion is governed by Laplacian potentials ϕ_0 and ϕ_1. The position of the ship geometry moving in the upper fluid, with speed U along the x_1-direction, is determined by $y = h_0 + \delta(\mathbf{x}, t)$ where $\delta(\mathbf{x}, t)$ determines the hull shape. The boundary condition at the ship geometry is given by $\delta_t + W_F = 0$ where $W_F = \mathbf{U} \cdot \nabla\delta$. The boundary of the upper fluid is denoted by F and is represented by a rigid lid at positions not occupied by the ship.

The interface, denoted by I, is determined by $y = \eta(\mathbf{x}, t)$. Values of the potentials along I are introduced by $\phi_{0I}(\mathbf{x}, t) = \phi_0(\mathbf{x}, y = \eta, t)$ and $\phi_{1I}(\mathbf{x}, t) = \phi_1(\mathbf{x}, y = \eta, t)$ on I, where indexes $0I$ and $1I$ indicate evaluation on the upper and lower side of the interface, respectively. Difference and sum potentials along I are introduced, where

$$\Psi(\mathbf{x}, t) = \phi_{1I}(\mathbf{x}, t) - \mu\phi_{0I}(\mathbf{x}, t) \quad \text{and} \quad \Phi(\mathbf{x}, t) = \phi_{0I}(\mathbf{x}, t) + \phi_{1I}(\mathbf{x}, t) \quad \text{at} \quad I, \tag{1}$$

and $\mu = \rho_0/\rho_1$. The interfacial motion and potential Ψ along I are integrated forward in time using the kinematic and dynamic boundary conditions at the interface:

$$\eta_t = V_I = W_I, \quad \Psi_t + g(1-\mu)\eta = \mathcal{NL}_2 \quad \text{at} \quad I, \tag{2}$$

where $\mathcal{NL}_2$ accounts for the full nonlinearity, $W_I = (\partial\phi_0/\partial n)\sqrt{1+|\nabla\eta|^2}$ and $V_I = (\partial\phi_1/\partial n)$ $\sqrt{1+|\nabla\eta|^2}$. Solution of the Laplace equation in each layer is obtained by use of Green's theorem. W_I, V_I and ϕ_{0F}, where the latter denotes the potential along the upper boundary F of fluid 0, including the ship surface, are expanded by $W_I = W_I^{(1)} + W_I^{(2)} + ...$, $V_I = V_I^{(1)} + V_I^{(2)} + ...$, $\phi_{0F} = \phi_{0F}^{(1)} + \phi_{0F}^{(2)} + ...$ In Grue (2015) it is shown that the quadratic approximation, i.e. truncating after the leading two terms of the expansions, fully accounts for the interfacial nonlinearity, for excursions η corresponding to the thinner layer depth. This approximation is used here.

Two- and three-layer fluids. The dispersion relation for a two-layer fluid reads $(ck_1)^2 = g(1-\mu)k_1/[\mu\coth(k_1h_0) + \coth(k_1h_1)]$, where k_1 denotes wavenumber and $c(k_1)$ wave speed (figure 1b). The dispersion relation for a three-layer fluid, where a pycnocline of thickness γ separates an upper mixed layer from a lower mixed layer, reads $K_\gamma^2 - k_1[\coth(k_1H_0) + \coth(k_1H_1)]K_\gamma\cot(K_\gamma\gamma) - k_1^2\coth(k_1H_0)\coth(k_1H_1) = 0$, where $K_\gamma^2 = N_0^2/c^2 - k_1^2$ and $N_0^2 = -(g/\rho)(\partial\rho/\partial y)$ denotes the buoyancy frequency, assumed to be constant in the three-layer model, see figure 1a, where also symbols are defined. The limit $k_1 \to 0$ obtains the linear long wave speed c_0. The wave phase speed $c(k_1)/c_0$ and group velocity $c_g(k_1)/c_0$ for the two- and three-layer fluids show small differences for kh_0 up to 0.7 (figure 1c).

Wave patterns and amplitudes. Ship geometries have a hull shape of $\delta = -b_0[1 - (x_1/(l_0/2))^6 - (x_2/(w_0/2))^6]$, where (l_0, w_0, b_0) denotes (length,width,draught). Calculations with $l_0/h_0 = 14.7, w_0/h_0 = 4$ and $b_0/h_0 = 1.2$ obtain the wave pattern for supercritical flow at speed $Fr = U/c_0 = 8$ (figure 2a). The pattern corresponds excellent to the patterns obtained by a linear kinematics analysis, for two- and three-layer fluids (Keller and Munk, 1970). Nonlinear calculations of the wave trough amplitudes show a small leading trough. Wave troughs number 2, 3, 4 and 5 have a common amplitude of $\sim 0.1h_0$, for a lateral distance of $x_2 \sim 100h_0$. The wave troughs follow power laws, i.e. $\eta_{min} = \beta x_2^{-\alpha}$ (figure 2b).

Other wake properties. A model similar to one of the ships in Watson et al. (1992) has a length of $38h_0$, width of $6.5h_0$ and draught of h_0. A mid depth of the pycnocline, in the field experiment, can be estimated to $h_0 = 3.06$ m. Calculations show the nonlinear interfacial depression and elevation below and right behind the ship (figure 3a,b), and the ship wake (figure 3c). Watson et al. showed measurements, obtained at an off-track distance of $55h_0$, of the cross-track current, u_2, and the strain rate, $\partial u_2/\partial x_2$, both at the sea surface, obtaining u_2/c_0 up to about $\sim \pm 0.1$, and $(\partial u_2/\partial x_2)/(c_0/h_0)$ up to about $\sim \pm 0.05$. The present calculations are very similar (figure 3d-f). In the simulations, wave tanks of lengths/widths of 300/300, 700/260, 500/400, have $\Delta x_1/h_0 \sim 0.4 - 0.8$ and $\Delta x_2/h_0 \sim 0.3 - 0.4$.

References.

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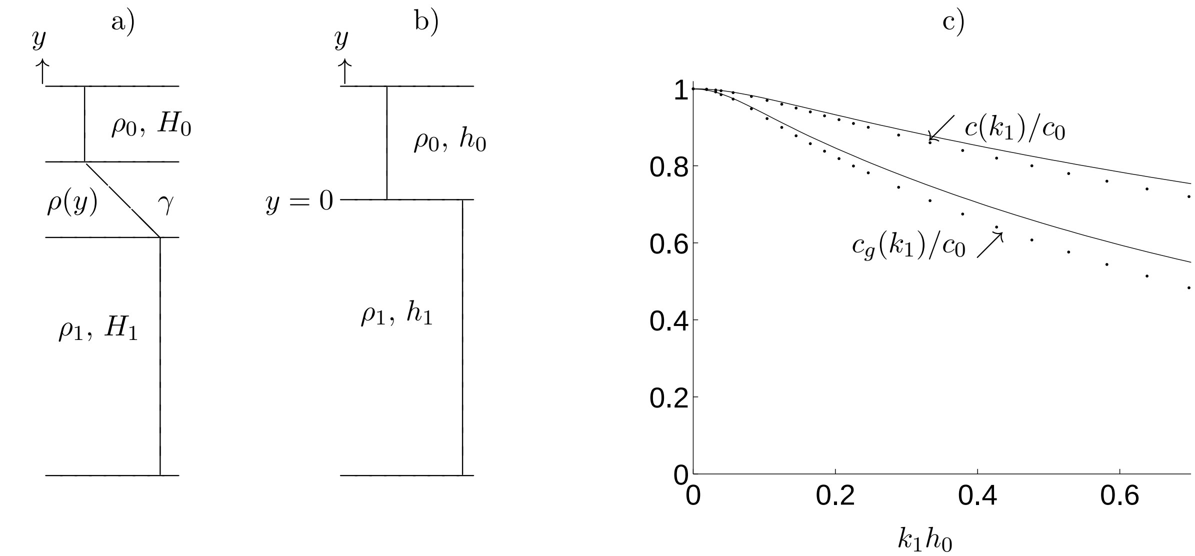

Figure 1: a) Sketch of theoretical three-layer fluid. b) Two-layer fluid. c) Wave phase speed $c(k_1)$ and group velocity $c_g(k_1)$, both normalized by the linear long wave speed of the respective density profiles. Two-layer configuration with $h_1/h_0 = 18$ (solid line) and three-layer configuration with $H_1/H_0 = 27.5$, $\gamma/H_0 = 1$ (dotted line).

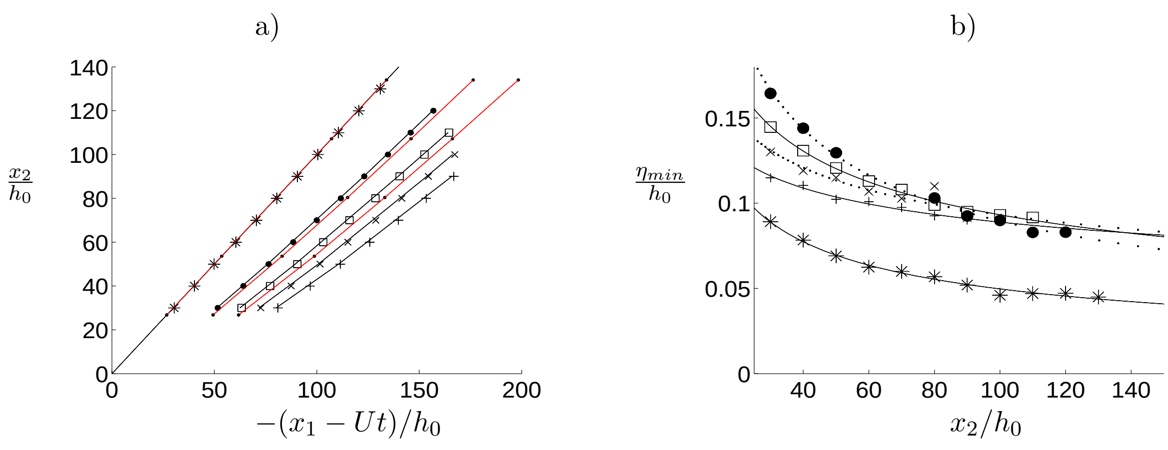

Figure 2: a) Nonlinear two-layer calculation of wake trough pattern (symbols); linear two-layer kinematics model with Keller-Munk equations (black solid line); linear three-layer kinematics model with Keller-Munk equations (red solid line). b) Trough amplitudes: trough 1 ($*$), trough 2 ($\bullet$), trough 3 (square), trough 4 ($\times$), trough 5 ($+$). $Fr = 8$. Ship model with length $29.4h_0$, width $4h_0$, draught $1.2h_0$.

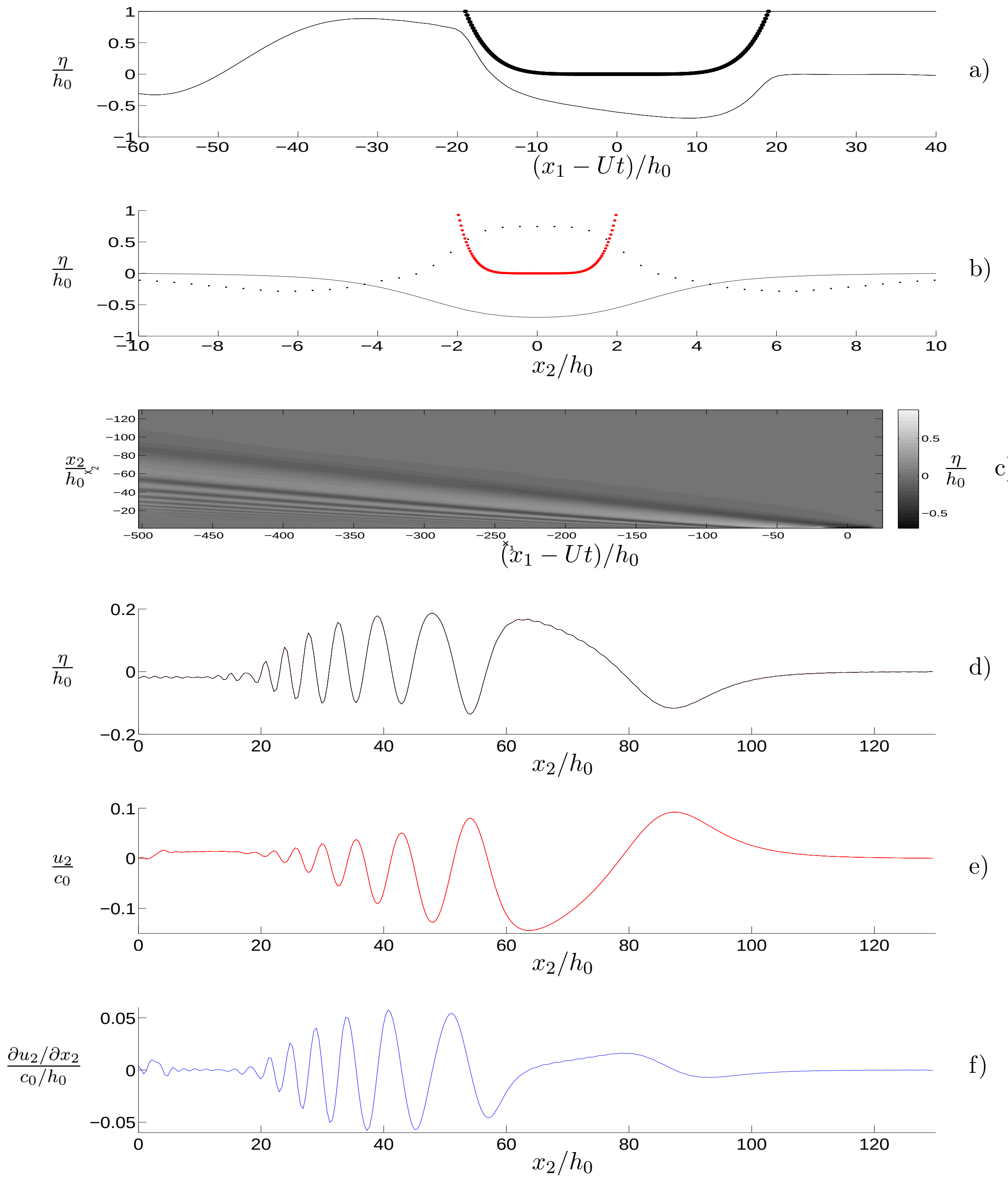

Figure 3: Interfacial wake properties for ship at $Fr = 6$. a) elevation η and hull geometry along centerplane of ship; b) η at two lateral cuts, at $(x_1 - Ut)/h_0 = 10$ (solid line) and $(x_1 - Ut)/h_0 = -30$ (dots) with ship geometry at mid beam (red solid line); c) $\eta(x_1 - Ut, x_2)$; d) cross-wake elevation η for $(x_1 - Ut)/h_0 = -504$; e) cross-wake velocity u_2 for $(x_1 - Ut)/h_0 = -504$; f) strain-rate $\partial u_2/\partial x_2$ for $(x_1 - Ut)/h_0 = -504$. Ship model with length $38h_0$, width $6.5h_0$, draught $1h_0$.