Towards Efficient Generalized Wagner Solvers for Slamming in Oblique Seas

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- For beam-sea calculations a simplification of the Generalized Wagner formulation is presented.
- The asymptotic corner flow model for beam sea is different from the head sea model.

1 Background and motivation

Ships operating in steep sea states may encounter critical loads due to wave slamming. In order to account for slamming loads in design, it is important to consider the frequency of occurrence for various load levels. As a consequence it is paramount to establish fast solvers for assessing a large number of impacts scenarios. The Generalized Wagner model first presented by Zhao et al. (1996) has previously been validated as a practical method in this context. Various formulations of the Generalized Wagner model has been discussed by Korobkin (2004). Based on full scale observations the largest slamming loads often occur in bow quartering seas. Most theoretical studies in this field have been focused on vertical impact of symmetric bodies. To some degree also for vertical impact of non-symmetric objects. Recently Lauzon (2014) presented a desingularized boundary element method for dealing with non-vertical impact velocity.

Towards impact simulation in oblique seas we will develop an efficient two-dimensional Generalized Wagner model applicable for beam sea calculations. The method is based on the meshless method presented by Helmers & Skeie (2013). In order to study important differences compared to the classical vertical drop tests, pure sway velocity in calm water is considered. By superposition of harmonic solutions this method can be combined with the theory of vertical impact models in order to handle general impact velocities. In the present study we only consider the spatial problem.

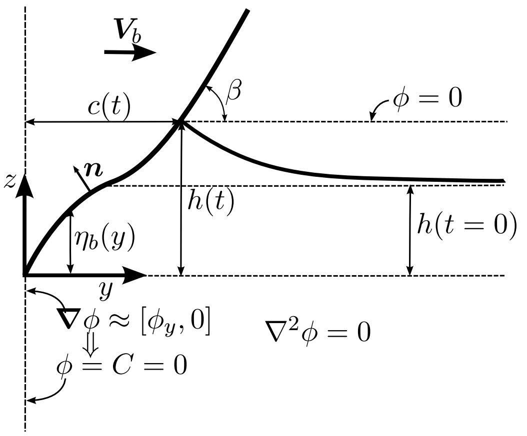
The target of this development is to establish a method suitable to be included in time domain seakeeping programs. Without loss of accuracy the theory is designed for exploiting preprocessor capabilities for efficient time domain calculations. As a consequence the spatial flow field needs to be accurately described by a very small number of parameters.

2 The spatial boundary value problem

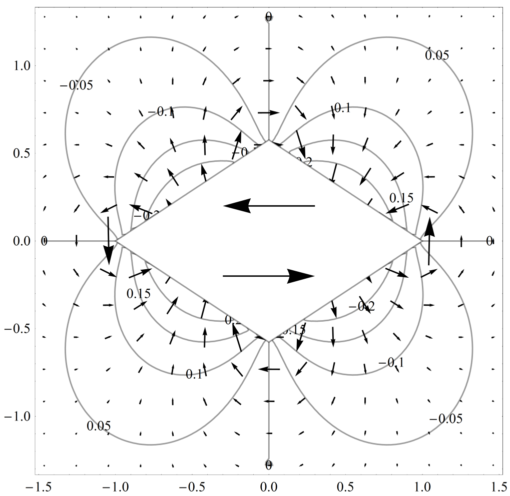
Horizontal motion of a two-dimensional surface piercing body in calm water is considered. A Cartesian coordinate system (y, z) is introduced. The origin is located at the keel and the y-axis is parallel with the undisturbed free surface. The z-axis is pointing upwards. The body contour is defined by $\eta_b(y)$.

$$z = \eta_b(y), \qquad \eta_b(0) = 0, \qquad \eta_b(-y) = \eta_b(y), \qquad 0 < \frac{\partial \eta_b}{\partial y}|_{y>0} < \infty, \qquad |y| \leq y_{\max} \tag{1}$$

The fluid flow is assumed ideal and described by the velocity potential ϕ. The horizontal body velocity $\boldsymbol{V}_b$ is assumed large in the sense that gravity will not affect the flow. Adopted from Zhao et al. (1996)



(a) The body velocity $\boldsymbol{V}_b$ is horizontal and the y-axis is parallel to the undisturbed free surface. At time t the intersection point between the body and the free surface is located at $y = c(t)$ and $z = h(t)$. On $y = 0$ we assume that the vertical fluid velocity is negligible compared to the horizontal component. The local deadrise angle at the intersection point is denoted β.



(b) The analytical flow field corresponding to the boundary value problem (2)-(4) induced by unit sway velocity of a wedge with $\beta = 30°$. The field vectors represent particle velocities and the curves are equipotential lines. The lower half is the physical space.

Figure 1: The boundary value formulation. The analytical solution for a wedge $\beta = 30°$.

the free surface flow is approximated by assuming a constant potential $\phi = 0$ on the horizontal line through the wave-body intersecting point as illustrated in figure 1a.

As a consequence of the impulsive nature of the flow and the pure horizontal body motion we will assume that the vertical fluid velocity at $y = 0$ is negligible compared to the horizontal component. The accuracy of this assumption is expected to be within the accuracy of neglecting horizontal fluid velocities on the free surface. It follows that ϕ is constant along the z-axis. By considering the assumption $\phi \to 0$ infinitely far away from the body we assume that $\phi = 0$ on $y = 0$. The spatial boundary value problem can be summarized as

$$\nabla^2\phi = 0, \qquad \text{in the fluid domain} \tag{2}$$

$$\boldsymbol{\nabla}\phi \cdot \boldsymbol{n} = \boldsymbol{V}_b \cdot \boldsymbol{n}, \qquad \text{on } z = \eta_b(y) \wedge y \leq c \tag{3}$$

$$\phi = 0, \quad \text{on } z = h \wedge y \geq c, \qquad \phi = 0, \quad \text{on } y = 0 \wedge z \leq 0, \qquad |\boldsymbol{\nabla}\phi| \to 0, \quad y^2 + z^2 \to \infty \tag{4}$$

where $\boldsymbol{n}$ is the unit surface normal pointing out of the fluid domain. The intersection point between the body surface and the true free surface is denoted $(y, z) = (c, h)$.

The meshless formulation (Helmers & Skeie 2013) is based on detailed knowledge of the flow characteristics. As a consequence we first need to consider the asymptotic properties of the boundary value problem (2)-(4).

Because of boundary conditions (4) it follows that the flow field far away from the body is represented by a quadrupole located at $(y, z) = (0, h)$ with unknown strength C_0.

$$\phi(y, z + h) = V_b \frac{C_0 yz}{(y^2 + z^2)^2}, \qquad y^2 + z^2 \to \infty \tag{5}$$

The decay rate of ϕ far away from the body is significant higher than for the vertical impact problem which is described by a vertical dipole. In the far field of the sway problem we note that the vertical fluid velocity is $\phi_z = C_0/y^3$ on the free surface.

The asymptotic flow close to the corners of the body is not evident for the sway problem. For the vertical impact problem success has been demonstrated by (e.g. Zhao et al. (1996)) using the classical corner flow formulation $\phi \sim r^{\sigma_1}$ where r is the asymptotic small distance between the corner and the field point. For the free surface intersection point the geometrical parameter σ_1 is related to the local deadrise angle β as $\sigma_1 = \frac{\pi/2}{\pi-\beta}$.

However, for sway motion of a floating circle with center on the free surface it is straight forward to derive a closed form expression for the global spatial potential. It is well known that the asymptotic expansion of that solution close to the free surface contact point is $\phi \sim r\ln(r)$. The cylinder is wall sided ($\sigma_1 = 1$) at this location. Hence the model applied by Zhao et al. (1996) is not capable of describing this actual corner flow.

In order to establish a relevant corner flow model valid for sway motion for all bodies we first study the analytical solution of boundary value problem (2)-(4) in case of a wedge with deadrise β. We apply the following conformal mapping relating the body surface to a unit circle

$$A\left(\frac{Z}{c}-1\right) = \int_1^W \left(1+\frac{1}{w^2}\right)\left(\frac{w^2+1}{w^2-1}\right)^{-2\nu} dw \tag{6}$$

$$A = \frac{2\sin(\pi\nu)}{\sqrt{\pi}}\Gamma(1-\nu)\Gamma(\frac{1}{2}+\nu) \in [0,2], \qquad \nu = \frac{1}{2\sigma_1}-\frac{1}{2} \in [0,\frac{1}{2}], \qquad \sigma_1 = \frac{\pi/2}{\pi-\beta} \in [\frac{1}{2},1] \tag{7}$$

where the physical location $Z = y + i(h+z)$ is mapped to the position W. It can then be shown that

$$\phi(Z) = \mathfrak{Im}\{\sum_{n=1}^{\infty}\frac{b_n}{W^{2n}}\}, \qquad b_n = \frac{4hV_b\cos\beta}{\pi A}\cdot I_n^\nu, \qquad h = c\tan\beta, \qquad i = \sqrt{-1} \tag{8}$$

$$I_n^\nu = \frac{1}{n}\int_{3\pi/2}^{2\pi}\cos\theta(\tan^2\theta)^\nu \sin 2n\theta\, d\theta = (-1)^n\frac{\Gamma\left(\frac{3}{2}-\nu\right)\Gamma(n+\nu)}{\Gamma\left(\frac{3}{2}+n\right)}S_n \tag{9}$$

$$S_n = {}_3F_2\left((\frac{1}{2}-n, 1-n, \frac{3}{2}-\nu), (\frac{3}{2}, 1-n-\nu), 1\right) \tag{10}$$

This flow field is plotted in figure 1b for $\beta = 30°$. $\Gamma(x)$ is the Euler Gamma function. For some deadrise angles, $\beta \in \{0°, 45°, 90°\}$, the summation in $\phi(Z)$ can be expressed by closed-form expressions. For other deadrise angles the evaluation of the generalized hypergeometric functions ${}_3F_2$ needs to be carried out using asymptotic relations for high values of n.

The important thing to note in our context is that for all β we can establish closed-form expressions for the asymptotic flow close to the corners derived from equation (8). At the free surface intersection the flow is characterized by a polylogarithm function of order $2+2\nu$ which close to the corner takes the form of $F_\beta(r,\alpha)$:

$$\frac{\phi(Z)}{hV_b} = c_1\left(\frac{r}{h}\right)^{\sigma_1}\sin(\sigma_1\alpha) + F_\beta(r,\alpha), \qquad Z = c + ih + r\exp(i\alpha), \qquad r \to 0 \tag{11}$$

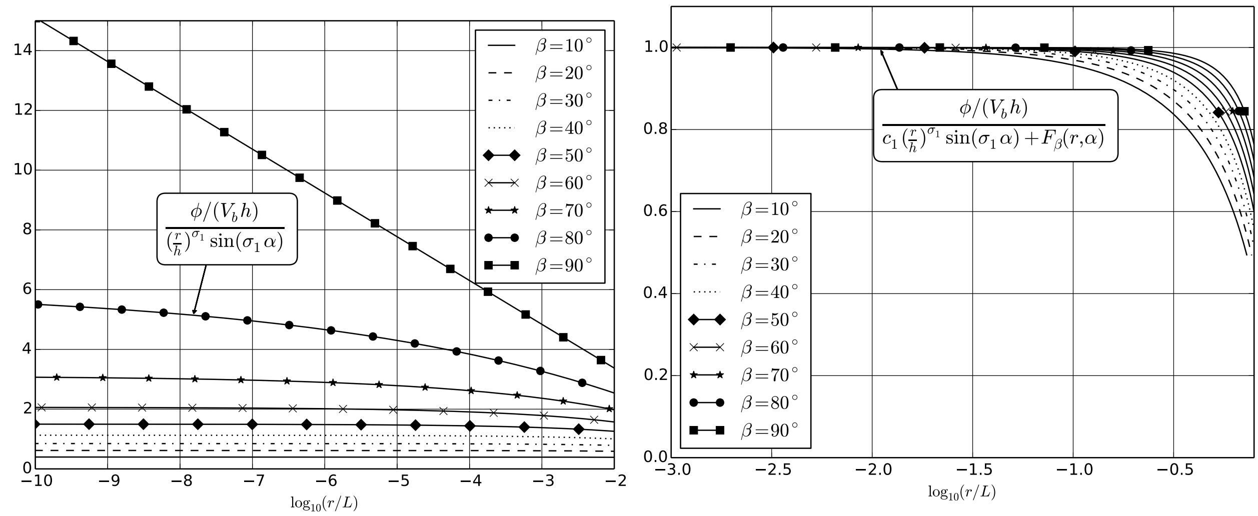
$$F_\beta(r,\alpha) = \tan(\beta)\left(-\frac{r}{h}\sin\alpha + c_2\left(\frac{r}{h}\right)^{\sigma_1}\sin(\sigma_1\alpha)\right) \tag{12}$$

$$c_2 = 2\,\zeta(1+2\nu)\frac{\sin(\pi\nu)}{\pi}\frac{\Gamma(1+\nu)}{\Gamma(1-\nu)}\left(\frac{2\cos(\pi\nu)}{\sqrt{\pi}}\Gamma(1-\nu)\Gamma(\frac{3}{2}+\nu)\right)^{\sigma_1} \tag{13}$$

where $\zeta(x)$ is the Riemann zeta function. A premature conclusion would be to include the c_2 term into c_1 and to neglect the only "body normal velocity" term $\tan(\beta)\frac{r}{h}\sin\alpha$ as a higher order effect ($\sigma_1 < 1$). The practical consequences of the latter strategy is shown in figure 2. It should be noted from our closed form expression for $F_\beta(r,\alpha)$ that for wall sided bodies $c_2 = 1$ and $F_\beta(r,\alpha) = \infty\cdot 0$. However by series expansion of $F_\beta(r,\alpha)$ it follows that

$$\lim_{\beta\to 90°} F_\beta(r,\alpha) = \frac{2}{\pi}\frac{r}{h}\left((1-\ln 2)\sin\alpha - \alpha\cos\alpha - \ln(\frac{r}{h})\sin\alpha\right) \tag{14}$$

which reflect the $r\ln r$ asymptotic nature related to wall sided bodies. An important observation from corner flow (11) is that there are no scaling factors related to $F_\beta(r,\alpha)$. The first term inside



(a) The classical corner flow model is only applicable for small β in case of sway. For $\beta = 80°$ the model is not adequate even as close to the corner as $r = 10^{-10}L$.

(b) For all β the proposed corner flow model approximate the flow excellent for the upper 1% of the wetted surface. For large β the model describes the flow reasonable good for the upper 20% of the wetted surface.

Figure 2: The applicability range for the classical and proposed corner flow models along the body is presented ($\alpha = \beta - \pi$). At the distance r from the free surface corner relative to the wetted lenght L between the corner and the keel, the corner flow model is applicable if the ratio between the true potential ϕ and the corner flow model is constant (flat curve). ϕ is calculated by the analytical model given by equation (8). In combination with discrete boundary element methods the corner flow model should at least be applicable for $\log_{10}(r/L) \approx -3$ in order to avoid huge discretization cost.

$F_\beta(r, \alpha)$ match exactly the local body boundary condition and should not be scaled even for non-wedge bodies. For all other flow terms, at any expansion order, $\partial\phi/\partial n = 0$ on the body. For curved bodies geometrical corrections of c_2 can be exactly replaced by a corresponding adjustment of c_1 in equation (11). As a consequence we will keep the corner flow formulation (11) for general geometries.

At the keel the corner flow is more similar to the classical corner flow because the *horizontal* body velocity is less strenuous on the local $\phi = 0$ condition than at the free surface. A local expansion of flow field (8) at the keel reveals

$$\frac{\phi(Z)}{hV_b} = c_3\left(\frac{r}{h}\right)^{\sigma_2}\sin\left(\sigma_2\left(\alpha+\frac{\pi}{2}\right)\right)+\frac{r}{h}\cos\alpha, \quad \sigma_2 = \frac{\pi}{\pi+2\beta}, \quad Z = r\exp(i\alpha), \quad r\to 0 \tag{15}$$

3 Conclusion and Further Work

A simplified spatial model for sway motion has been presented. The analytical results are now being implemented in the meshless Generalized Wagner formulation. A temporal model will follow.

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