Study on a Semi-Analytic Approach for Analysis of Parametric Roll in Regular and Irregular Head Seas

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Highlights:

- A semi-analytic approach for the simulation of parametric roll motions is proposed from the investigation on the characteristics of metacentric height (GM) variation and the relationship between GM and the restoring lever (GZ) in waves.
- To validate the accuracy and efficiency of the present method, the computational results are compared with those obtained by the direct simulation based on an impulse response function (IRF) method.
- The stochastic properties of parametric roll in irregular waves are discussed, considering the uncertainties due to the computational parameters such as time-window and the phases of wave components.

1. Introduction

Parametric roll motion in head sea is a rapidly increasing large-amplitude motion induced by the variations of restoring forces over several wave periods. Due to the nonlinear change in transverse stability, there is a difficulty in quantifying the amplitude of parametric roll. Also, this phenomenon in irregular waves is a non-ergodic and non-Gaussian process, which leads to the large number of realizations to obtain the stable probabilistic qualities. Therefore, an efficient and accurate scheme which can consider the nonlinearities in parametric roll is required for the practical application.

In a time-domain numerical computation, the nonlinear restoring lever in waves can be considered in a more accurate manner for the quantitative calculation. Shin et al. (2004) applied the Rankine panel method to develop the susceptibility criteria for parametric roll motions of large container ships while Spanos and Papanikolaou (2007) used IRF method to analyze the phenomenon of fishing vessels in regular waves. However, large computational costs are incurred for many realizations and for many sea states in wave scatter diagram. On the other hand, Bulian (2004) and Umeda et al. (2004) derived the simplified analytical approaches for prediction of amplitude of parametric roll by applying their own modellings for the nonlinearities in the variations in GZ. Since then, led by the International Maritime Organization (IMO), various models such as Belenky's, Umeda's, and Song's methods (IMO, 2010, 2011, 2012) have been developed to establish dynamic stability criteria and operational guidance, but the accuracies of these conventional methods are still concern.

In this study, a semi-analytical approach to simulate the parametric roll motion of a large container ship in head sea conditions is proposed. The accuracy and efficiency of the present approach are validated by the comparison with the results obtained by the IRF method in respect of the occurrence and amplitude of parametric roll in regular and irregular waves. Also, the probabilistic qualities of the parametric rolling in irregular waves as obtained from the both a semi-analytical and numerical approach are compared considering the sensitivities and uncertainties of to the computation parameters.

2. Semi-analytic approach

Based on the assumption that the coupling effects from roll motion to vertical motions are neglected, the 1.5 degree of freedom (DOF) equation of roll motion in head seas can be written as follows:

$$\left(M_{44}+A_{44}\right)\ddot{\xi}_4+b_{roll}\dot{\xi}_4+\Delta GZ\left(\xi_3,\xi_4,\xi_5,\eta\right)=0 \tag{1}$$

where M_{44}, A_{44}, and Δ are the rolling moment of inertia, the roll- added moment of inertia, and the displacement, respectively. In Eq. (1), the damping force acting on roll motion is modelled by an equivalent linear damping to for an easy implementation. Also, the restoring lever, GZ in the equation depends on the actual wetted surface considering vertical and roll motions of ship relative to the wave elevation. Because large computation costs are incurred for the direct integration of the external pressure up to wave surfaces in time domain, the simplified and efficient modelling for the time-varying restring force is required.

2.1. Approximation of GM

Containerships which have the large bow and overhang transom show a large variation in the water plane area according to the change in the draft at the fore and after body. The resulting variations of GM in waves can be approximated by the mean value and harmonic components, such that:

$$GM(A,\omega,t) = GM_{still} + GM_0(A,\omega) + \sum_{i=1}^{\infty} GM_i(A,\omega)\cos(i\omega t) \approx GM_{still} + GM_0(A,\omega) + GM_1(A,\omega)\cos(\omega t) \quad (2)$$

where GM_{still} indicates the GM in still water, and GM_0 and GM_i are the difference between the mean and GM_{still}, and the i-th harmonic component, respectively. The time history of GM is calculated at the actual water plane area considering vertical motions obtained by the IRF method. According to Eq. (2), the transfer functions of GM_0 and GM_1 are computed by applying the Fourier transform to the time histories of GM in regular waves as shown in Fig. 1. The higher order harmonic components are assumed to be negligible since these values are relatively small and are not directly related with the mechanism of occurrence of parametric roll.

In this study, GM_0 which represents the static stability performance is regarded as being a second order quantity with respect to the wave amplitude based on the assumption that the second order effects on the mean value are dominant. On the other hand, GM_1 which is the amplitude of variation is a linear component as proven by Dunwoody (1989). It can be seen that these hypotheses are valid by the fact that the normalized transfer functions computed for different wave amplitudes show good correspondences between each other. Therefore, the variations of GM in random seas can be approximated under the "assumption of superposition", such that:

$$GM_0(H_s,T_p) = \sum_{i=1}^{N} A_i^2 RAO_{GM_0/A^2}(\omega_i) = 2\int_0^{\infty} RAO_{GM_0/A^2} S_\eta(\omega)d\omega = 2\int_0^{\infty} S_{GM_0/A^2}(\omega)d\omega \quad (3)$$

$$GM_1(H_s,T_p,t) = \sum_{i=1}^{N} A_i RAO_{GM_1/A}(\omega_i)\cos(\omega_i t + \varepsilon_i) \quad (4)$$

where $RAO_{GM0/A2}$ and $RAO_{GM1/A}$ are the transfer functions of GM_0 and GM_1, respectively. S_η and $S_{GM0/A2}$ indicate the spectrum of the wave and normalized GM_0, respectively.

2.2. Approximation of GZ

The variation in GZ is proportional to the variation in GM at small heel angles while the variation shows nonlinearities from the body geometry due to the small change in the wetted surface at large heel angles. Therefore, the GZ in waves is expressed by the GM_{still}, GM in waves, and the "GZ factor function", $f(\xi_4)$ as follows:

$$GZ(\xi_4,t) = GZ_{still}(\xi_4) + \frac{GM(t) - GM_{still}}{GM_{still}} f(\xi_4). \quad (5)$$

The accuracy of quantitative prediction is closely relevant to how to model the higher order term in the GZ factor function to consider the nonlinearities at large heel angles. Eq. (6) denotes the present modeling which is a modified form of the Song's method (IMO, 2012), such that:

$$f(\xi_4) = GM_{still}\left(\sin(\xi_4) - \frac{\sin^{\alpha}(\xi_4)}{\sin^{\alpha-1}(\xi_{4,\max})}\right) \quad (6)$$

where $\xi_{4,\max}$ indicates the x-intercept of the GZ curve in still water. In the current method, α is adopted to represent the order of the higher-order term.

The proposed GZ factor function is compared with that from the "direct calculation" which is based on the instantaneous hydrostatically computed maximum and minimum GZ curves in waves as shown in Fig. 2. For an appropriate value of α, the present function shows good agreement with that of the direct calculation in overall heel angles. It should be noted that the validated values are different according to the type of ships (for the 6500 TEU containership, α=2.0-3.0, for the MARIN model 8004-2, α=5.0-7.0), which means that α depends on the geometry of ship, especially at fore and after body.

3. Parametric roll in regular waves

The equation of roll motion in a regular wave can be solved by using the time integration such as the 4th-order Runge-Kutta method. In the integration, the disturbance of roll motion induced by gusts or currents is modelled by an impulsive roll angle (3-10 degrees) at a certain instant. After parametric roll arises, the roll motion does not diverge, and is bounded with the quasi-steady state amplitude owing to the damping force and the decreases of GZ in waves at large heel angles.

Fig. 3 shows the comparison of the quasi-steady state amplitudes obtained by the present method and the IRF method which is based on the hydrodynamic coefficients of the strip theory and the weakly nonlinear approach. The discrepancies with regard to the amplitudes and occurrences of parametric roll come from the difference between the calculation methods for the hydrodynamic forces and the errors in the approximation of variations in GZ at large heel angels. For the validated value of α, however, the similar solutions are produced by the semi-analytical and the numerical method.

4. Parametric roll in irregular waves

The solutions of equation of roll motion in random seas exhibit the stochastic nature such as non-Gaussianities and non-ergodicities because of the nonlinear restoring term which can be regarded as a random process. In the solutions, there exist strong uncertainties and sensitivities with respect to the length of time window, the method of discretization for the wave spectrum. Therefore, the stochastic analysis instead of the deterministic calculation is required to investigate the statistical properties of parametric roll in irregular waves.

The test condition is the "Run 307004" case (*Tp*=14.4 sec, *Hs*=5.25 m) from the benchmark study conducted by the ITTC specialist committee on stability in waves (Reed, 2011). The randomly discretized 80 wave components for a wave spectrum are given, and the 20 sets of wave phases are distributed to represent the wave train of the experiment in the study. For these wave conditions, it was proved that parametric roll occurs easily. In the both applications of the semi-analytical and numerical approach, the same impulsive angle is imposed, and the damping coefficient is set based on the roll decay test of the study. Also, α of the present method is in the range of 2.0-3.0 according to the validation for the model ship, the MARIN model 8004-2.

For all 20 realizations in which the 2500-sec simulations are conducted, the variances of parametric roll motions and their 95% confidence bands are computed as shown in Fig. 4. The IRF method shows the more scattered variances along with larger bands. This phenomenon indicates that the results of numerical approach are more sensitive to the phases of wave components than those of the semi-analytic approach because the higher-order components in nonlinear restoring forces and Froude-Krylov forces are taken into account based on weakly nonlinear approach. On the other hand, the simplified variations of GM in the present method lead to the more consistent results for different set of wave phases. If the value of α, which denotes how large the variations of restoring forces in waves increases, the variations and the uncertainties becomes large owing to the stronger nonlinearities in the equation of roll motion.

Fig. 5 shows the cumulative density functions (CDFs) of parametric roll motions for different time windows. Even for the 86400-sec simulation, the numerical method exhibits still diverged CDFs (the absence of the "practical ergodicities") while the semi-analytical method show the converged functions which has the similar form with the Rayleigh distribution. For larger value of α, the differences from the Rayleigh distribution increase with the stronger non-Gaussianities, as expected. Despite the more accurate considerations for the nonlinear restoring forces in the numerical computation, in conclus*i*on, the present method can be a more rational way because the converged statistical properties can obtained much more efficiently.

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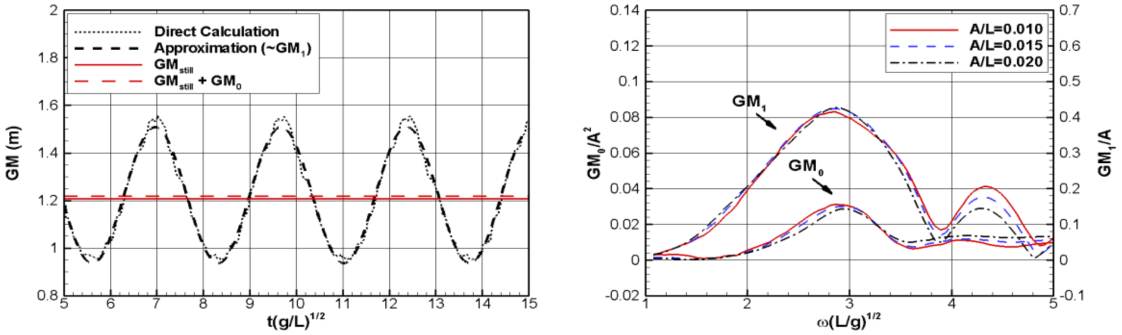
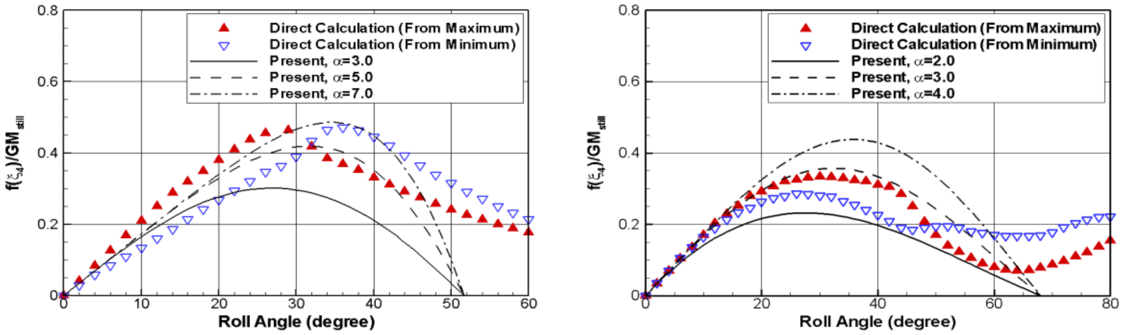
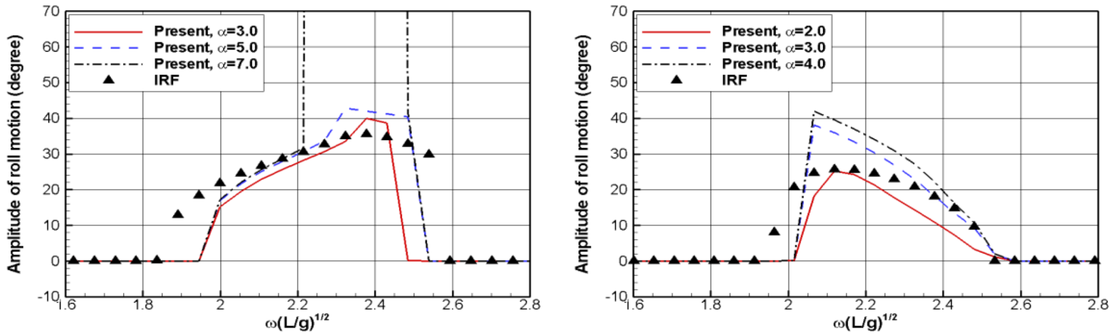


Fig. 1 Time history of GM in a regular wave and its approximation, $\omega(L/g)^{1/2}$=2.11 (left), Transfer functions of GM (right): 6500 TEU containership, *V* (forward speed)=5 knots



(a) 6500 TEU containership, $\omega(L/g)^{1/2}$=2.11 (b) MARIN model 8004-2, $\omega(L/g)^{1/2}$=2.27

Fig, 2 GZ factor functions: A/L=0.010, V=5 knots



(a) 6500 TEU containership (b) MARIN model 8004-2

Fig, 3 Quasi-steady state amplitude of parametric roll motion: A/L=0.010, V=5 knots

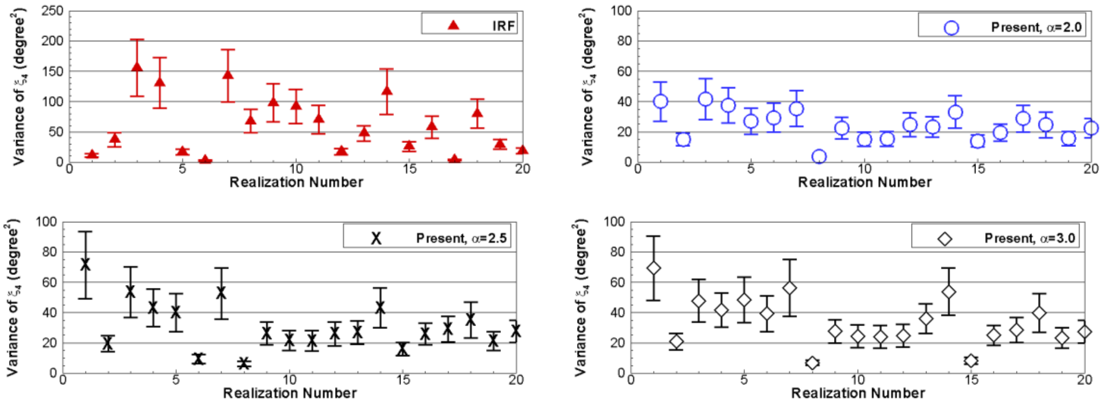


Fig. 4 Variances of parametric roll with 95% confidence bands: Run 307004 (Reed, 2011), 2500 sec

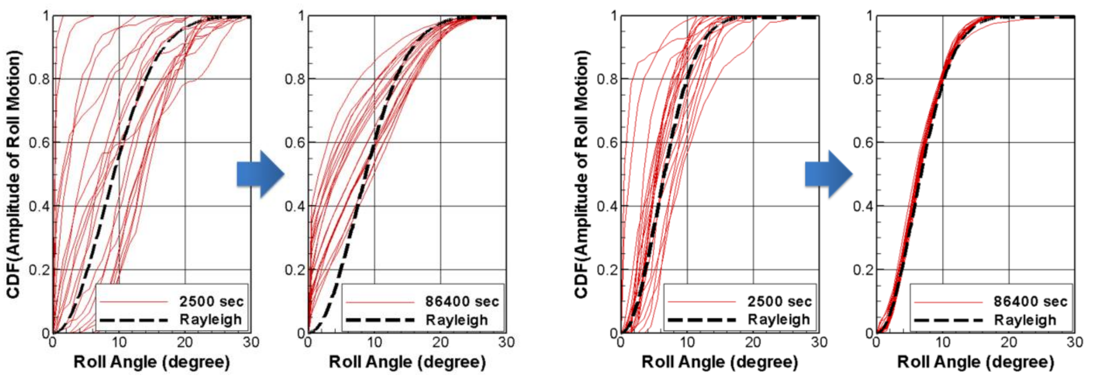


Fig. 5 Cumulative density functions: Run 307004 (Reed, 2011), IRF method (left), Present, α=2.5 (right)