A fully submerged flap-type wave energy converter

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Highlights:

• A novel solution method for the diffraction and radiation of waves by a fully submerged flap is presented.

• Insight is given into the effects on performance seen as a result of fully submerging the device.

1. Introduction

The abundance of energy available in ocean waves has long been of interest as a potentially significant source of renewable energy, numerous devices having been conceived over the years with the intent of converting it into a usable form. However, difficulties arise in engineering devices which are both efficient and able to survive the harsh marine environment. One of the devices which has recently emerged as a promising candidate for large scale commercial success in the UK is the Oyster device, a buoyant flap-type device under development by Aquamarine Power LTD[1]. Although engineering development challenges remain this has been successfully demonstrated in terms of numerical modelling, laboratory testing and in full-scale deployment. Thus, interest has turned to mathematically modelling devices of this type with a significant contribution having been made by Renzi & Dias (2013).

With the issue of survivability firmly in mind, the purpose of the present paper is to investigate the impact on the performance characteristics of such devices when they are fully submerged and thus sheltered from the most extreme conditions which are seen in the surface region. To that end a novel semi-analytic solution method is developed.

2. Formulation

Cartesian coordinates have been chosen with the origin at the mean free surface level and z pointing vertically upwards. The fluid has density ρ and is of constant, finite depth h. The hydrodynamic model assumes the flap to be infinitely thin and buoyant so that when at rest it occupies the vertical plane $\{x = 0, -a < y < a, -h < z < -h + b\}$, where $b < h$. It is hinged along a horizontal axis $(x, z) = (0, -h + c)$, which is denoted in figure 1 by P. Above its pivot the flap is free to move and below it is held fixed and vertical. The fluid is incompressible and inviscid, the flow

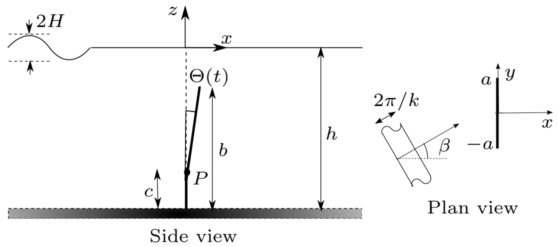

Figure 1: Side and plan views of the flap converter used in the hydrodynamic model.

is irrotational and the flap oscillations are assumed to be of small amplitude. A standard linearised theory of water waves is used.

Small amplitude plane waves of radian frequency ω are incident from $x < 0$, making an anti-clockwise angle $\beta \in (-\pi/2, \pi/2)$ with the positive x-direction.

After removing the harmonic time-dependence, the velocity potential is decomposed as

$$\phi(x, y, z) = A\phi^S(x, y, z) + \Omega\phi^R(x, y, z) \qquad (1)$$

where Ω is the complex angular velocity, $A = igH/2\omega\psi_0(0)$ ensures an incident wave height H and ψ_0 is a normalised depth eigenfunction which will be defined later. Here ϕ^S and ϕ^R are associated with the scattered and radiated wave fields respectively. They satisfy

$$\nabla^2\phi^{S,R} = 0 \qquad (2)$$

in the fluid,

$$\phi_z^{S,R} - \frac{\omega^2}{g}\phi^{S,R} = 0 \qquad (3)$$

on $z = 0$, and

$$\phi_z^S = 0 \qquad \text{and} \qquad \phi_z^R = 0 \qquad (4)$$

on $z = -h$. Further, we have

$$\phi_x^S\left(0^\pm, z\right) = 0 \qquad \text{and} \qquad \phi_x^R\left(0^\pm, z\right) = u(z) \qquad (5)$$

for $-a < y < a$ and $-h < z < -h + b$, where

$$u(z) = \begin{cases} 0, & z \in [-h, -h+c] \cup [-h+b, 0] \\ z + h - c, & z \in [-h+c, -h+b]. \end{cases} \qquad (6)$$

[1] http://www.aquamarinepower.com/

The incident wave is given by

$$\phi^I(x,y,z) = e^{ik(x\cos\beta - y\sin\beta)}\psi_0(z) \tag{7}$$

where k satisfies $\omega^2 = gk\tanh kh$ and $\psi_0(z)$ is a normalised depth eigenfunction defined by

$$\psi_0(z) = N_0^{-1/2}\cosh k(z+h), \tag{8}$$

with $N_0 = \frac{1}{2}(1+\sinh(2kh)/(2kh))$. The potentials ϕ^R and $\phi^D \equiv \phi^S - \phi^I$ describe outgoing waves at large distances from the flap.

In the frequency domain, the equation of motion of the flap is

$$-i\omega I\Omega = -\frac{iC}{\omega}\Omega + F_w + F_e \tag{9}$$

where I denotes the moment of inertia and C the restoring moment, quantities determined by the physical properties of the flap. F_w is the time-independent wave torque and is written as

$$F_w = AF_S + \Omega F_R \tag{10}$$

where

$$F_{S,R} = -i\omega\rho\int_{-a}^{a}\int_{-h}^{0} P^{S,R}(y,z)u(z)\,dz\,dy \tag{11}$$

and

$$P^{S,R}(y,z) = \phi^{S,R}\left(0^+,y,z\right) - \phi^{S,R}\left(0^-,y,z\right) \tag{12}$$

is the pressure difference across $x=0$. Further decomposition of the radiation force F_R yields

$$F_w = AF_S + (i\omega\mathcal{A} - \mathcal{B})\,\Omega \tag{13}$$

where the real quantities $\mathcal{A}(\omega)$ and $\mathcal{B}(\omega)$ are the added mass and radiation damping coefficients. Finally, we decide the mechanism for power take-off should take the form of a linear damping force and write

$$F_e = -\lambda\Omega \tag{14}$$

where λ is assumed to be a real constant so that the power and velocity are in phase. It may then be shown (see Evans & Porter (2012) for example) that the capture factor can be written as

$$\hat{l} = \hat{l}_{max}\frac{2\mathcal{B}}{\mathcal{B}+|Z|}\left(1 - \frac{(\lambda-|Z|)^2}{|\lambda+Z|^2}\right) \tag{15}$$

where

$$\hat{l}_{max} = \frac{1}{2a}\frac{|AF_S|^2}{8\mathcal{B}W_{inc}} \tag{16}$$

is the maximum capture factor, achieved through optimal tuning ($\lambda = |Z|$) at resonance ($|Z| = \mathcal{B}$), W_{inc} is the power per unit width of incident wave and

$$Z = \mathcal{B} - i\omega\left(\mathcal{A} + I - \frac{1}{\omega^2}C\right). \tag{17}$$

Further, the optimal capture factor is given by

$$\hat{l}_{opt} = \frac{2\mathcal{B}}{\mathcal{B}+|Z|}\hat{l}_{max} \tag{18}$$

and is achieved through optimal tuning.

Thus, in order to study the performance of the device we must first determine the hydrodynamic coefficients $\mathcal{A}$ and $\mathcal{B}$ along with the exciting force F_S. These depend on the solution of the hydrodynamic problems for ϕ^S and ϕ^R and that is where our attention turns now.

3. Solution of the hydrodynamic problems

3.1 The scattering problem

The scattering problem deals with the diffraction of the incident wave when the flap is held fixed vertically. We consider the potential $\phi^D \equiv \phi^S - \phi^I$ associated with the diffracted waves. By antisymmetry we have $\phi^D(x,y,z) = -\phi^D(-x,y,z)$ and so we only need the solution in $x > 0$. We define the Fourier transform of $\phi^D(x,y,z)$ by

$$\overline{\phi}^D(x,l,z) = \int_{-\infty}^{\infty}\phi^D(x,y,z)e^{-ily}\,dy. \tag{19}$$

Then, taking Fourier transforms with respect to y of the governing Laplace equation (2) gives

$$\left(\nabla_{xz}^2 - l^2\right)\overline{\phi}^D = 0. \tag{20}$$

The most general solution of (20) which also satisfies (3) and (4) with the correct outgoing wave behaviour is

$$\overline{\phi}^D(x,z) = \sum_{r=0}^{\infty} B_r(l)e^{-\lambda_r x}\psi_r(z) \tag{21}$$

where $B_r(l)$ are unknown coefficients,

$$\psi_r(z) = N_r^{-1/2}\cos k_r(z+h), \tag{22}$$

$N_r = \frac{1}{2}(1+\sin(2k_rh)/(2k_rh))$ and k_r are the positive roots of $\omega^2 = -gk_r\tan k_rh$ for $r = 1,2,\ldots$. This is consistent with the definition of $\psi_0(z)$ if we let $k_0 = -ik$ and the functions $\psi_r(z)$ for $r = 0,1,2,\ldots$ form a complete set of normalised depth eigenfunctions. Further,

$$\lambda_r(l,k_r) = \begin{cases} \left(k_r^2 + l^2\right)^{1/2}, & \text{for } r = 1,2,\ldots \\ \left(l^2 - k^2\right)^{1/2}, & \text{for } r = 0 \text{ and } |l| \geq k \\ -i\left(k^2 - l^2\right)^{1/2}, & \text{for } r = 0 \text{ and } |l| < k \end{cases}$$

where the choice of branch for λ_0 ensures the radiation condition is satisfied.

We formulate the problem in terms of the unknown pressure difference across the flap, defined in (12). Taking Fourier transforms and using the orthogonality of the depth eigenfunctions we gain the following equations for the unknown coefficients

$$B_r(l) = \frac{1}{2h}\int_{-h}^{-h+b} \overline{P}^S(l,z')\psi_r(z')dz' \equiv \frac{1}{2h}\int_{-h}^{-h+b}\int_{-a}^{a} P^S(y',z')\psi_r(z')e^{-ily'}dy'\,dz' \quad (23)$$

for $r = 0, 1, 2, \ldots$.

Invoking the inverse Fourier transform of (21) results in an integral representation for $\phi^D(x,y,z)$

$$\phi^D(x,y,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\sum_{r=0}^{\infty} B_r(l)\,e^{-\lambda_r x}\psi_r\,(z)\,e^{ily}\,dl \quad (24)$$

where $B_r(l)$ for $r = 0, 1, 2, \ldots$ are expressed in terms of $P^S(y',z')$ in (23). Applying the condition on the flap,

$$\frac{\partial\phi^D}{\partial x}(0^{\pm},y,z) = -\frac{\partial\phi^I}{\partial x}(0^{\pm},y,z) \quad (25)$$

for $-a < y < a$ and $-h < z < -h+b$, then results in an integral equation for $P^S(y,z)$. This may not be solved analytically, instead we employ a Galerkin expansion method. We incorporate the known square-root endpoint behaviour through the approximation

$$P^S(y,z) \simeq \sum_{n=0}^{2N+1}\sum_{p=0}^{P}\alpha_{np}w_n(y)\tau_p(z) \quad (26)$$

where

$$w_n(y) = \frac{e^{in\pi/2}}{a(n+1)}\sqrt{a^2-y^2}\,U_n\left(\frac{y}{a}\right) \quad (27)$$

and

$$\tau_p(z) = \frac{2e^{ip\pi}}{\pi b(2p+1)}\sqrt{b^2-(z+h)^2}\,U_{2p}\left(\frac{z+h}{b}\right) \quad (28)$$

and $U_n(\cos\theta) = \sin((n+1)\theta)/\sin\theta$ are Chebyshev polynomials of the second kind. Substituting for $P^S(y',z')$ in the integral equation, multiplying through by $-(1/\pi)w_m^*(y)\tau_q(z)$ and integrating over $-a < y < a$, $-h < z < -h+b$ results in the following system of linear equations

$$\sum_{n=0}^{2N+1}\sum_{p=0}^{P}\alpha_{np}M_{npmq} = D_m(\beta)G_{q0} \quad (29)$$

for $m = 0, \ldots, 2N+1$ and $q = 0, \ldots, P$, where

$$M_{npmq} = \sum_{r=0}^{\infty} G_{pr}G_{qr}K_{nm}^{(r)} \quad (30)$$

with

$$K_{nm}^{(r)} = \frac{1}{4}\int_{-\infty}^{\infty}\frac{\lambda_r(l,k_r)}{l^2}J_{n+1}(al)J_{m+1}(al)\,dl \quad (31)$$

and

$$G_{pr} = \begin{cases} N_r^{-1/2}J_{2p+1}(k_r b)/k_r h & \text{for } r \geq 1 \\ (-1)^p N_0^{-1/2}I_{2p+1}(kb)/kh & \text{for } r = 0 \end{cases} \quad (32)$$

and

$$D_m(\beta) = \begin{cases} -i\cot\beta J_{m+1}(ka\sin\beta) & \text{if } \beta \neq 0 \\ -\frac{1}{2}ika\delta_{m0} & \text{if } \beta = 0. \end{cases} \quad (33)$$

The integrals which determine $K_{nm}^{(r)}$ vanish when $n+m$ is odd, a rendundancy which allows us to reduce our consideration to elements for which $n+m$ is even. In order to ensure rapid convergence we use an integral result involving products of Bessel functions (Gradshteyn & Ryzhik (1981) §6.538(2)) to gain an integrand which decays like $O\left((ka)^2/l^4\right)$. Ultimately (29) then reduces to a coupled pair of systems which may be solved for the unknown expansion coefficients α_{np}.

The exciting torque on the flap may be expressed in terms of the Galerkin expansion coefficients as

$$F_S = \frac{1}{2}i\omega\rho a h^2\pi\sum_{p=0}^{P}\alpha_{0p}\hat{g}_p \quad (34)$$

where

$$\hat{g}_p = -\int_{-h}^{0}\tau_p(z)u(z)dz, \quad (35)$$

an integral which may be expressed in closed form.

3.2 The radiation problem

Applying the same solution method to the radiation problem, this time making the approximation

$$P^R(y,z) \simeq ah\sum_{n=0}^{2N+1}\sum_{p=0}^{P}\beta_{np}w_n(y)\tau_p(z) \quad (36)$$

for the unknown pressure difference across the flap, results in the following system of linear equations

$$\sum_{n=0}^{2N+1}\sum_{p=0}^{P}\beta_{np}M_{npmq} = E_m\hat{g}_q \quad (37)$$

for $m = 0, \ldots, 2N+1$ and $q = 0, \ldots, P$. Here M_{npmq} is defined identically to before,

$$E_m = \frac{1}{2}\delta_{m0} \quad (38)$$

and $\hat{g}_q$ is given in (35).

More rapid convergence of the integrals defining $K_{nm}^{(r)}$ may be achieved as before. Ultimately, having solved for the unknown expansion coefficients β_{np}, we find that the radiation torque is given by

$$F_R = -\frac{1}{2}i\omega\rho a^2h^3\pi\sum_{p=0}^{P}\beta_{0p}\hat{g}_p. \quad (39)$$

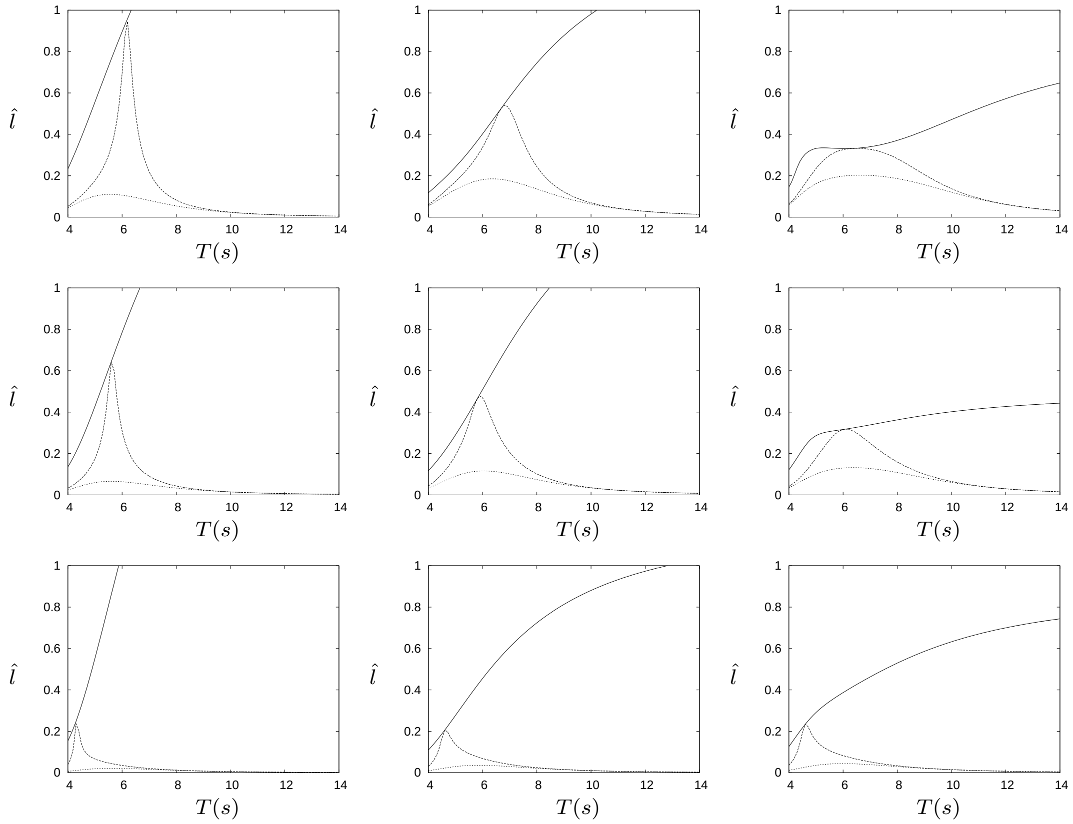

Figure 2: Capture factors plotted as a function of wave period $T(s)$ for flaps of various lengths and heights in water of fixed depth $h = 12m$. The hinge height is fixed at $c = 0.2h$ and the power take-off at $\lambda = 8$. The rows show results for $b/h = 0.9, 0.8$ and 0.6 respectively, moving down the page, whilst the columns show results for $a/h = 0.5, 1.0$ and 2.0, from left to right. The dotted, dashed and solid curves show $\hat{l}$, $\hat{l}_{opt}$ and $\hat{l}_{max}$ respectively.

4. Results

Figure 2 shows the actual, optimum and maximum capture factors for a range of flap lengths and proportions of the depth taken up by the device. The results appear to be best when $b/h = 0.9$ and the top of the flap is nearest to the surface. The theoretical maximum, which forms an upper bound, is at its highest when $a/h = 0.5$ and the flap is short. However, this also corresponds to a narrower resonant peak than that seen for longer devices. Whilst an improvement in the actual capture factors over those plotted may be achieved through optimal tuning, this unfortunate combination of characteristics leads to a narrow peak and actual capture factors being limited to a mean value of about 0.3. By comparison, when optimally configured, the results for a surface piercing device are close to 0.7 for a broad range of periods (Noad & Porter, 2015). It is not obvious that such a deterioration in performance should be seen as a result of complete submersion. Indeed, high capture factors are maintained for some devices and this is the case for the fully submerged device of Crowley, Porter & Evans (2014) for example.

References

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