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Fully Nonlinear Computations of Wave Radiation Forces and Hydrodynamic Coefficients for a Ship with a Forward Speed

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1 Introduction

The accurate prediction of nonlinear wave loads on a ship with a forward speed is critical to the evaluation of it's global motion performance and manoeuvring capabilities. The seakeeping performance is one of the key factors for hull form optimization. Traditionally, the seakeeping computation can be performed using the strip theory developed, for example, by Salvesen *et al.* (1970) or the one extended to the body-exact version in time-domain for large amplitude motion computations by Zhang *et al.* (2010*a*). Recently, with rapid development of computing hardware, three-dimensional time-domain approaches using Rankine sources or desingularized sources have been developed to accurately model and capture the forward speed effects. Due to the nonlinear properties of both the free surface kinematic and dynamic boundary conditions (FSBC) applied on the unknown free surface, the FSBC and body boundary condition (BBC) are often linearized with respect to the calm water surface at $z = 0$ (z is the vertical coordinate). Different forms of linearizations have been developed over the past several decades, including Neumann-Kelvin linearization (NKL) and double body linearization (DBL). The comparisons of those linearized approaches to a time-domain body exact strip theory and experiments have been extensively studied in a previous paper (Zhang *et al.*, 2010*b*).

However, due to the assumptions made upon deriving the linearized FSBC and BBC, both NKL and DBL may have limited validity depending on Froude number or slenderness of a hull. The details on the NKL and DBL models are presented in the next Section. Without any assumptions on small parameters, fully nonlinear computations may provide more reliable solutions and can be employed as a benchmark for the linearized or quasi-nonlinear models.

The primary focus of the present study is on identifying and quantifying the nonlinearities associated with wave-body interaction including forward speed and hull slenderness. We developed a fully nonlinear model to compute the wave radiation forces on vessels travelling with a forward speed. The final objective is to quantify validity of different linearized models.

2 Theory and Approach

2.1 Fully nonlinear model

We developed a fully nonlinear potential flow computational model to study the wave radiation problem for ships with a forward speed. Desingularized sources and Rankine panels are applied on the free surface and instantaneous wetted hull surface, respectively. On the free surface, a mixed Euler-Lagrange free surface tracking scheme is employed (Longuet-Higgins & Cokelet, 1976; Yeung, 1982; Zhang *et al.*, 2010*a*,*b*). The nonlinearities associated with both the body boundary condition and the free surface boundary conditions (except wave breaking) are automatically accounted for in the developed three-dimensional time-domain model.

The vessel is assumed to move with speed $\mathbf{U}(t) = (U_o(t), 0, 0)$, and may be undergoing unsteady oscillations in six degrees of freedom. The fluid is assumed to be ideal and the flow irrotational. Three coordinate systems are employed: the $\mathbf{x_o}$ system is fixed in space, the $\mathbf{x}$ system is fixed to the mean position of the ship (moving with forward speed $\mathbf{U}(t)$ along the straight track of the ship), and the $\overline{\mathbf{x}}$ system is fixed to the ship. The boundary value problem is solved in the right hand moving coordinate system (x, y, z), as illustrated in Figure 1. The x-axis points in the direction of travel and the z-axis points upward. The origin is on the calm water plane at midship.

In the $\mathbf{x}$ coordinate system, a velocity potential is introduced to describe the fluid motion by using the above assumptions such that the fluid velocity can be expressed as the gradient of a potential function,

$$\mathbf{V}(\mathbf{x}, t) = \nabla\Phi = \nabla(-U_o(t)x + \phi(x, y, z, t)) \tag{1}$$

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where ϕ is the disturbance velocity potential which may include the radiation and/or diffraction potential.

The velocity potential $\phi(x, y, z, t)$ satisfies the Laplace equation

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \tag{2}$$

The exact nonlinear kinematic and dynamic free surface boundary conditions are

$$\frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z} - \nabla\phi \cdot \nabla\eta + U_o(t)\frac{\partial \eta}{\partial x} \qquad \text{on} \qquad z = \eta(x, y, t) \tag{3}$$

$$\frac{\partial \phi}{\partial t} = -g\eta - \frac{1}{2}\nabla\phi \cdot \nabla\phi + U_o(t)\frac{\partial \phi}{\partial x} \qquad \text{on} \qquad z = \eta(x, y, t) \tag{4}$$

where $\eta(x, y, t)$ represents the free surface elevation; g is the gravitational acceleration. All the velocity potentials satisfy the Laplace equation under the assumption of ideal potential flow.

The exact body boundary condition can be written as

$$\mathbf{n} \cdot \nabla\phi = U_o(t)n_1 + \mathbf{V_H} \cdot \mathbf{n} - \nabla\phi^I \cdot \mathbf{n} \qquad \text{on} \quad S_B \tag{5}$$

where S_B is the instantaneous wetted body surface; $U_o(t)$ is the time-dependent translating velocity of the body in the x direction; $\mathbf{n}$ is the inward unit normal on the body surface(out of fluid); n_1 is the component of the unit normal in the x direction; $\mathbf{V_H}$ is the motion velocity including rotational modes of a point on the ship's surface; ϕ^I is the velocity potential for an incident wave .

The initial conditions at $t = 0$ are

$$\Phi = \Phi_t = 0 \quad \text{in the fluid domain} \tag{6}$$

At each time step, a mixed boundary value problem must be solved; the potential is given on the free surface and the normal derivative of the potential is known on the body surface. In order to solve the initial boundary value problem, desingularized sources are distributed over the free surface and constant strength flat panels are utilized on the body surface. Given the strength of the desingularized sources above the free surface and sources distributed on the body surface, the potential at any point in the fluid domain can be obtained. The details on the application of desigularized sources and panels have been presented in previous papers (Zhang *et al.*, 2010*a*,*b*).

The free surface is time stepped through applying a mixed Euler-Lagrange scheme on the fully nonlinear free surface boundary conditions (3) and (4). The wetted hull surface is re-panelized at each time step by updating the instantaneous intersection curve between the wetted hull and dynamic wave surface. After solving the boundary value problem at each time step, the wave radiation force and moments, and the hydrodynamic coefficients can be computed.

2.2 Neumann-Kelvin linearization (NKL)

The nonlinear free surface boundary conditions are often linearized by introducing a basis flow Ψ. The total perturbation velocity potential can be written as

$$\phi = \Psi + \phi^{'} \tag{7}$$

where $\Psi \sim O(1)$ and $\phi^{'} \sim O(\epsilon)$ are assumed.

In Neumann-Kelvin linearization, it is assumed that the uniform flow $-U_0(t)x$ in translating coordinate system $\mathbf{x}$ is not disturbed due to the presence of a hull, which leads to $\Psi = 0$. Linearized FSBC and BBC can be obtained by substitute $\phi \sim O(\epsilon)$ and $\eta \sim O(\epsilon)$ into Eqns. (3), (4) and (5).

2.3 Double-body basis flow linearization (DBL)

In double-body linearization, the basis flow is computed by assuming an unbounded uniform flow passing a double-body. Hence we have the following:

$$\partial\Psi/\partial z = 0 \quad \text{at} \quad z = 0 \tag{8}$$

$$\mathbf{n} \cdot \nabla\Psi = U_o(t)n_1 \quad \text{on} \quad S_{\bar{B}} \tag{9}$$

where $S_{\bar{B}}$ is the mean wetted body surface. A set of linearized FSBC and BBC can be derived through substituting Eqns. (7), (8) and (9) into Eqns. (3), (4) and (5) and keeping all the leading order terms. We remark the obtained linearized BBC contains so-called m-terms which represent the coupling effects between the steady flow and unsteady flow (see Zhang *et al.*, 2010*b*). For purpose of comparison, we also build up a

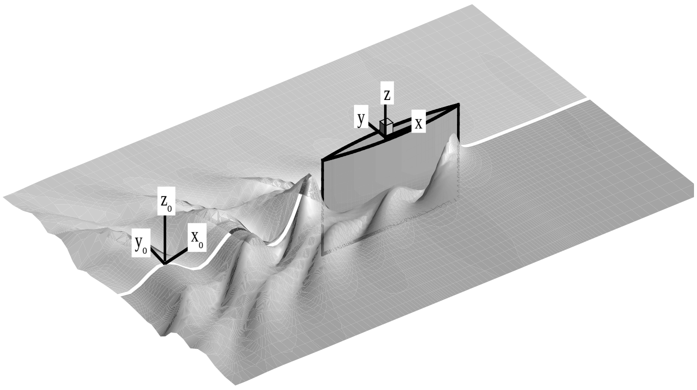

Figure 1: Definition of the problem and coordinate systems

model (called LFS DB m-terms) using the double-body m-terms in BBC while retaining the linearized free surface boundary conditions the same as the one in Neumann-Kelvin model.

It should be noted that wave breaking, which is a natural phenomenon, can occur in both wave radiation and diffraction problems for a ship travelling with a forward speed. The wave breaking normally can be observed near the bow, stern and in the wake. In the present study, we focus on non-wave breaking cases and keep the forward speed and the forced motion amplitude of hull relatively small to prevent the wave breaking in the simulations.

3 Results and Discussion

The developed fully nonlinear model is validated by comparing the obtained numerical solutions to experiments and other numerical results using different linearized models. The tested hull forms include Wigley I hull and a Series 60 hull with $C_B = 0.7$. The comparison of the diagonal/coupling added mass and damping coefficients due to a heave motion are illustrated in Fig.2. The forced motion frequencies $\omega\sqrt{L/g}$ vary from 2.2 to 4.5. The numerical solutions using fully nonlinear computations are compared with those using DBL, NKL, linearized FSBC with double-body m-terms, and experimental data reported by Journeé (1992). As can be seen from the figure, the present computations of added mass and damping agree quite well with experiments and show better agreement with measured data than other linearized models including NKL and linearized FSBC with double-body m-terms (LFS, DB m-terms). We remark here that the predictions using fully nonlinear simulations are very close to the solutions using DBL. This confirms that DBL is a valid linearized model for the Wigley I hull at $F_n = 0.3$. The computed hydrodynamic coefficients due to a forced pitch motion are illustrated in Fig.3. The same conclusion is found for the predicted hydrodynamic coefficients.

We are also investigating the dependence of the hydrodynamic coefficients on Froude number and the hull slenderness, for both a Wigley hull and a Series 60 hull. The additional results will be presented at the workshop.

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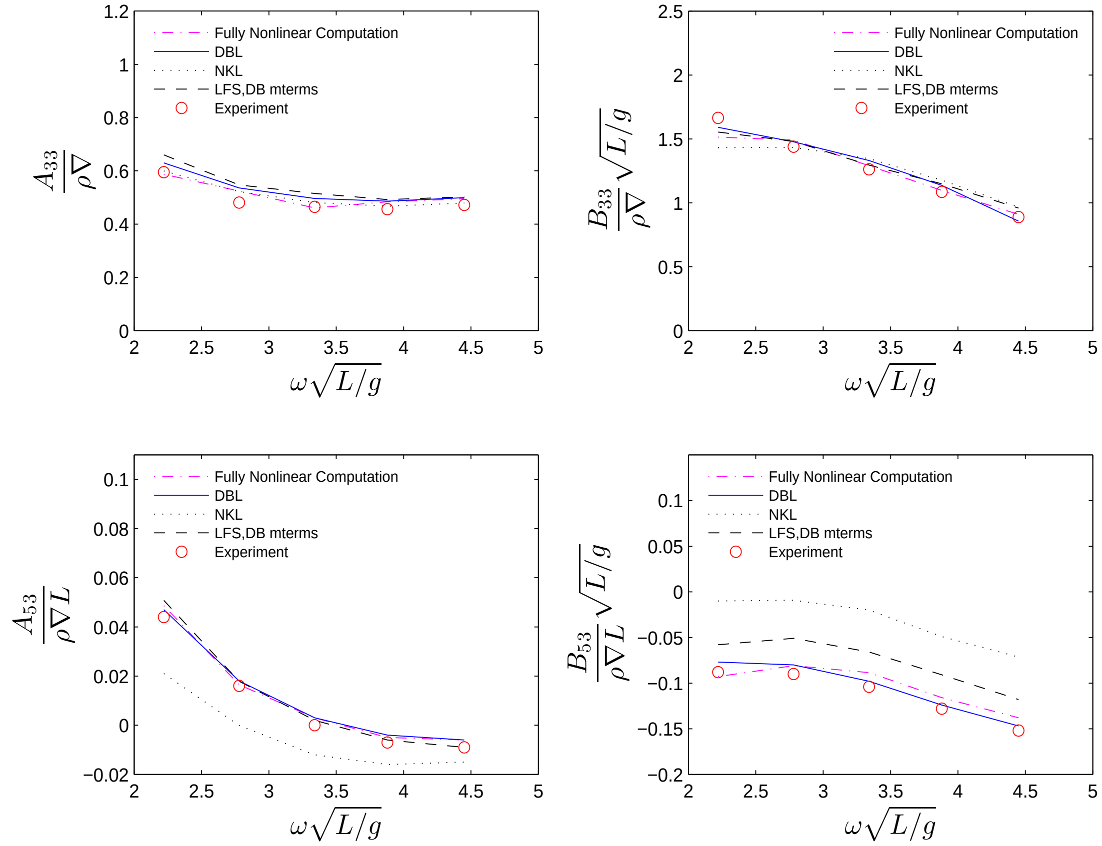

Figure 2: The hydrodynamic coefficients due to a forced heave motion for Wigey I hull, Froude number $F_n = 0.3$, heave motion amplitude $a/L = 0.01$, L is the length of the ship; ω is the forced motion frequency.

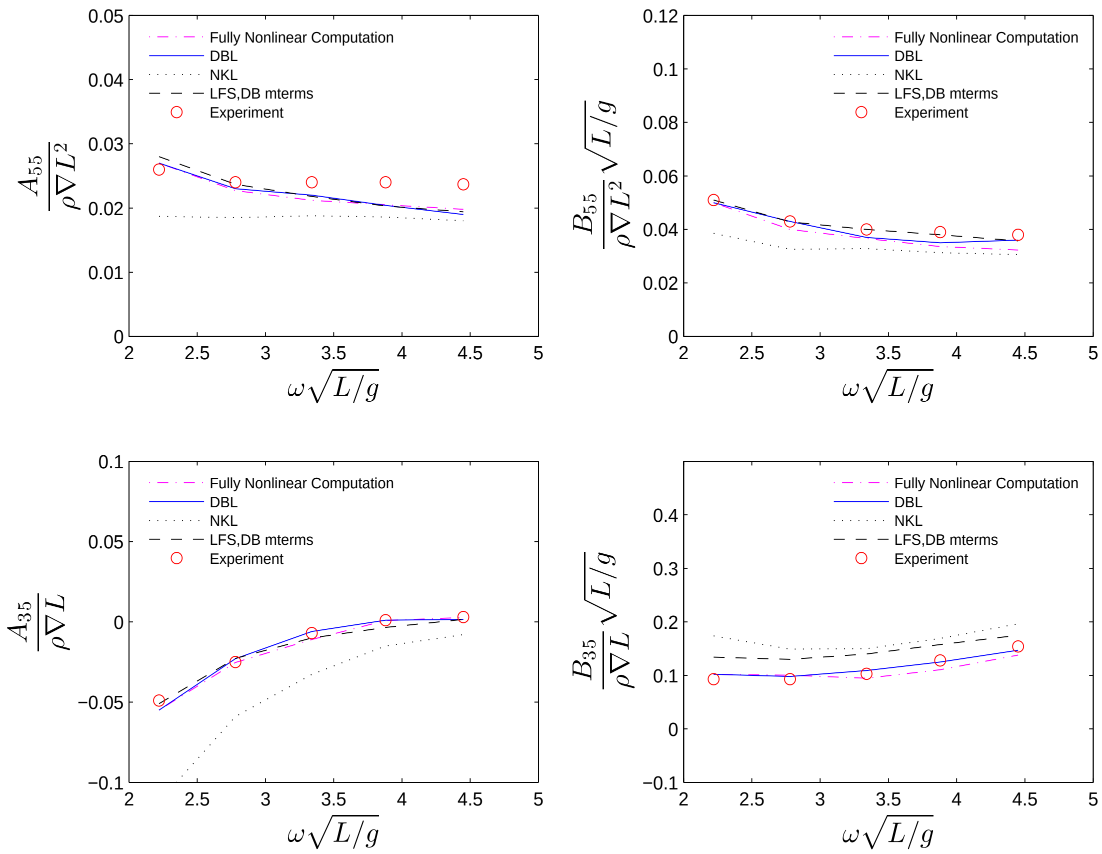

Figure 3: The hydrodynamic coefficients due to a forced pitch motion for Wigey I hull, Froude number $F_n = 0.3$, pitch motion amplitude $a = 0.05$ rad with $L = 10$ m.