MATH20901 Multivariable Calculus
Problems Class Week 2

1. Explain what is meant by a linear map. With $\mathbf{x} = (x_1, x_2, x_3)$ which of these are linear maps from $\mathbb{R}^3 \to \mathbb{R}^2$:

$$\text{(i) } \mathbf{f}(\mathbf{x}) = (x_2 + x_3, x_1 + x_2), \qquad \text{and} \qquad \text{(ii) } \mathbf{f}(\mathbf{x}) = (x_2 x_3, x_1 x_2).$$

   Explain why.

2. Let $\mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $\mathbf{F}(\mathbf{x}) = (x_2 x_3, x_1 x_2)$. Find the derivative of $\mathbf{F}$ in the direction $(1, -1, 1)$ using two independent methods.

3. Let

$$\mathbf{F}(u, v, w) = (v^2 + uw, u^2 + w^2, u^2 v - w^3),$$

   and

$$\mathbf{G}(x, y) = (xy^3, x^2 - y^2, 3x + 5y),$$

   and define $\mathbf{H}(x, y) = (\mathbf{F} \circ \mathbf{G})(x, y)$. Compute $\mathbf{H}'(-1, 1)$.

4. Given

$$z = f\left(\frac{x + y}{x - y}\right),$$

   show that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0.$$

5. Show that the pair of equations

$$x^2 - y^2 - u^3 + v^2 + 4 = 0, \qquad 2xy + y^2 - 2u^2 + 3v^4 + 8 = 0$$

   determine local functions $u(x, y)$ and $v(x, y)$ defined for (u, v) near $(u, v) = (2, 1)$ such that $(x, y) = (2, -1)$. Compute $\dfrac{\partial u}{\partial x}$ at $(x, y) = (2, -1)$, $(u, v) = (2, 1)$.