BrOx Titles and Abstracts (7th July, 2016)

Sam Chow, *Roth–Waring–Goldbach*

Abstract: We use Green's transference principle to show that any subset of the dth powers of primes with positive relative density contains nontrivial solutions to a translation-invariant linear equation in d^2+1 or more variables, with explicit quantitative bounds.

Ben Green, *Sarkozy's theorem in function fields*

Abstract: Sarkozy's theorem asserts that if $A \subset [N]$ is a set with density α then $A - A$ contains a kth power. In this theorem, α is allowed to tend to zero as $N \to \infty$. Sárközy's original work showed that a rate of decay $\alpha \sim (\log N)^{-c_k}$ is possible, but it is plausible that the theorem still holds if $\alpha \sim N^{-c'_k}$. This is an open problem, but it turns out that the recent breakthrough of Croot, Lev and Pach allows one to establish just such a statement in the function field model for the problem.

Sofia Lindqvist, *Partition regularity of generalised Fermat equations*

Abstract: Let a, b, c be integers, and consider the equation $x^a + y^b = z^c$. We show that any k-colouring of $\mathbb{F}_p$ has a monochromatic nontrivial solution to this equation, provided p is sufficiently large in terms of the number of colours and the exponents. This is new already in the case $a = b = 1$, $c = 2$, that is for the equation $x + y = z^2$. An outline of the proof is given in this special case, and it is based on techniques developed by Green and Sanders for proving partition regularity of configurations $\{x, y, x + y, xy\}$ over $\mathbb{F}_p$.

Hong Liu, *On the number of subsets of $[n]$ with no k-term arithmetic progression*

Abstract: Addressing a question of Cameron and Erdős, we show that, for infinitely many values of n, the number of subsets of $\{1, 2, \ldots, n\}$ that do not contain a k-term arithmetic progression is at most $2^{O(r_k(n))}$, where $r_k(n)$ is the maximum cardinality of a subset of $\{1, 2, \ldots, n\}$ without a k-term arithmetic progression. This bound is optimal up to a constant factor in the exponent. For all values of n, we prove a weaker bound, which is nevertheless sufficient to transfer the current best upper bound on $r_k(n)$ to the sparse random setting. To achieve these bounds, we establish a new supersaturation result, which roughly states that sets of size $\Theta(r_k(n))$ contain superlinearly many k-term arithmetic progressions. This is joint work with Jozsef Balogh and Maryam Sharifzadeh.

Trevor Wooley, *Efficient congruencing for deficient systems*

Abstract: We consider estimates stemming from the efficient congruencing method (or, equivalently, l^2-decoupling methods) for mean values of exponential sums over polynomials not necessarily of Vinogradov's type. By orthogonality, such mean values are related to systems of equations deficient relative to Vinogradov systems — there are missing equations, the presence of which would complete a translation-dilation invariant system.