Speakers and Abstracts.

Aleksi Pyörälä (Oulu)

Covering the Sierpinski carpet with tubes

A set on the plane is called tube-null if it can be covered by strips of arbitrarily small total width. This notion has its roots in harmonic analysis and motivates the following question, also interesting from a purely geometric point of view: Which kind of sets are tube-null?

While sets of small Hausdorff dimension are easily seen to be tube-null, the property is often difficult to verify for larger sets. Indeed, only a few non-trivial examples of tube-null sets of dimension larger than one seem to be known. I will present our recent result, joint with P. Shmerkin, V. Suomala and M. Wu, that closed sets invariant under multiplication by an integer (other than the torus) are tube-null. Included among these sets is the classical Sierpinski carpet.

Alec Chamberlain Cann (Bristol)

More refined multifractal spectra for Bedford-McMullen carpets

For a self-affine measure μ on a Bedford McMullen carpet, one can compute the multractal spectrum

$$f(\alpha) = \dim_{\mathcal{H}} X_\alpha$$

where $X_\alpha = \{x \in X : \text{loc}_\mu(x) = \alpha\}$ and $\text{loc}_\mu(x)$ is the local dimension at x. King computed this for carpets with a certain separation condition which was later lifted by Jordan and Rams.

We can also consider the set of points for which the local dimension does not exist, and consider for these points the value taken by the upper and lower local dimension. Let

$$Y_\alpha = \{x \in X : \underline{\text{loc}}_\mu(x) = \alpha\} \text{ and } Z_\alpha = \{x \in X : \overline{\text{loc}}_\mu(x) = \alpha\}$$

where $\underline{\text{loc}}_\mu(x)$ and $\overline{\text{loc}}_\mu(x)$ denote the upper and lower local dimension at x respectively. We then wish to understand $g(\alpha) = \dim_{\mathcal{H}} Y_\alpha$ and $h(\alpha) = \dim_{\mathcal{H}} Z_\alpha$. It is always true that $f(\alpha) \leq g(\alpha)$ and $f(\alpha) \leq h(\alpha)$, but we exhibit an example for which this inequality is strict and we get phase transitions.

Dániel Prokaj (Budapest)

Continuous Piecewise Linear Iterated Function Systems on the line

We study the dimension theory of Continuous Piecewise Linear Iterated Function Systems (CPLIFS) on the line. An iterated function system $\mathcal{F}$ is called CPLIFS if it consists of finitely many continuous, strictly contracting, piecewise linear self-mappings $\{f_1, \ldots, f_m\}$ of $\mathbf{R}$ with non-zero slopes. Opposed to the widely investigated IFS families, like linear or hyperbolic iterated function systems, a CPLIFS might contain non-injective mappings as well.

A natural upper bound on the Hausdorff dimension of the attractor is the root $s_{\mathcal{F}}$ of a naturally associated pressure function called non-additive upper capacity topological pressure (introduced by L. Barreira). We show that if the slopes of the functions of $\mathcal{F}$ are smaller than 1/3, or smaller than 1/2 assuming that all the functions are injective, then the same holds typically (in the sense of the packing dimension of the exceptional set) for overlapping constructions as well.

The new results of the talk are joint with Károly Simon.

Liam Stuart (St. Andrews)

Sullivan's dictionary and Assouad-type dimensions

Sullivan's dictionary is a framework used to study the relationships between two areas of mathematics, namely Kleinian groups and rational maps. The dictionary contains many analogous results with similar proofs, and one particularly strong correspondence comes from the perspective of dimension theory, where many notions of dimension are given by a 'critical exponent'. In this talk, I will discuss how by expanding the family of dimensions considered to those of Assouad-type, we can find a richer family of similarities between the two settings, and also some clear differences. Based on joint work with Jonathan Fraser.

Markus Myllyoja (Oulu)

Dimension of limsup sets of rectangles in Heisenberg groups

Dimensions of limsup sets have drawn a lot of interest in recent times and there are various instances where the almost sure value of the Hausdorff dimension of random limsup sets is known. In this talk I will discuss a recent result by F.Ekström, E.Järvenpää and M.Järvenpää where they obtained an almost sure dimension formula for Hausdorff dimension of limsup sets of rectangles in the first Heisenberg group. I will also talk about generalizations of this result to higher-dimensional Heisenberg groups and to other types of rectangles, which is the topic I am currently working on.

Desmond Li (Bristol)

Laplacians on Fractals

The classical spectral theory of Laplacian focuses only on smooth structures e.g. smooth surface, smooth manifolds, but never on fractals. Although in some cases, we might have a surface with a fractal boundary, say a filled Julia set, but the interior is always smooth. This motivates us to try and construct a Laplacian that extends the domain of definition to fractals as well. The hope is that, by studying spectral theory problems on fractals, we would shed some light to our original classical problems which might be difficult or even inaccessible to solve if we only restrict ourselves to a smooth definition. For my talk I will briefly explain the work of Kigami and Krein on their fractal Laplacian definition and various results. Then, I will talk about results I obtained by applying the definition and similar methods to circles which was not well studied.

Tianhong Yang (Bristol)

Projecting ergodic measure on the overlapping Sierpiński gasket

Questions on dimensions of projected measures have been long studied. It is well-known that, under open set condition, the dimension of self-similar measure is equal to similarity dimension which corresponds to the entropy of a Bernoulli measure defined on shift space over Lyapunov exponent. We focus on the case where we project an ergodic measure onto an overlapping Sierpinski gasket. To show the dimension of projected measure is equal to entropy over Lyapunov exponent for a very large set of contraction ratios, we study the cross sections by understanding results on generalized Bernoulli convolutions. Based on work supervised by Thomas Jordan.

Amlan Banaji (St. Andrews)

Dimensions of continued fraction sets

Fractal dimensions of sets of numbers which have real or complex continued fraction expansions with restricted entries have been extensively studied. We will explain that these sets are limit sets of infinite conformal iterated function systems and discuss what is known about the Hausdorff and box dimensions. The intermediate dimensions are a family of dimensions which interpolate between the Hausdorff and box dimensions, and we present formulae for the intermediate dimensions of such sets. We will consider applications to fractional Brownian images and general Holder images. Based on recent joint work with Jonathan Fraser.