Galois representations — Exercises to Lecture 2

**Exercise 1.** Show that if $\rho_E$ is irreducible then $E$ acquires good supersingular reduction over a finite extension. Recall that $E/K$ has good supersingular reduction if $\#\tilde{E}(\mathbb{F}_K) \equiv 1 \bmod p$.

**Exercise 2.** Let $K$ be a local field with residue characterisitic $p$. Let $A/K$ be a 3-dimensional abelian variety and $\rho_A : \mathrm{Gal}(\overline{K}/K) \to \mathrm{GL}_6(\overline{\mathbb{Q}}_\ell)$ be the $\ell$-adic representation attached[1] to $A$ , with $\ell \neq p$.
(i) Show that if $A/K$ has potentially good reduction then for every $i \in I_{\bar{K}/K}$ the order of $\rho_A(i)$ is either 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 18, 20, 24 or 30.
(ii) Do all of these occur? (This requires knowledge of curves and abelian varieties beyond the lecture.)

[1] i.e. on the dual of the Tate module or, equivalently, on $H^1_{et}(A/\bar{K}, \mathbb{Q}_\ell) \otimes \overline{\mathbb{Q}}_\ell$