Background on BSD and its statement over number fields:
[Birch1], [Birch2], [Tate-BSD]; Cassels papers on III [CasselsIV, CasselsIV] (+ its non-squareness for abelian varieties [PoonenStoll]); \(\zeta\) - and \(L\)-functions [Serre-ZetaL]; \(l\)-adic representations [Serre-AbLAdic].

Potential modularity & using Brauer/Solomon induction arguments to reduce to modular curves: the course followed [DD-Parity] §2; for background and proofs see [Taylor3] proof of Thm. 2.4, [Taylor4] proof of Cor. 2.2 and Wintenberger’s appendix to [Nek-SelIV].

BSD for rank \(\leq 1\):
Gross-Zagier formula [GrossZagier] ([Zhang, TianZhang] over totally real fields) + Kolyvagin’s Euler system argument [Kolyvagin] + existence of rank \(\leq 1\) quadratic twist [BFH], [MurtyMurty], [Waldspurger];
Parity predictions:
Twists of semistable curves [Rohrlich-Root, D-Root]; No local expression for the rank [DD-ModN]; Failure of Goldfeld over number fields [Connell, DD-Evil].

Galois action on the Tate module:
[SerreTate], [Serre-PropGal]; the inertia action for \(v\nmid 2, 3\) can be recovered from Tate’s algorithm [Tate-alg], for \(v\mid 2, 3\) see [Kraus, DD-Isogeny].

Weil-Deligne representations and root numbers:
Definitions, existence of \(\epsilon\)-factors and main properties [DeligneC, Tate-NThBg]; Frohlich-Queyrut theorem [FrohQuey]; Galois invariance of root numbers [Rohrlich-Inv]; see also summary in [DD-Regconst] Appendix A.

Root numbers of elliptic curves:
[Rohrlich-Root] (all cases except additive reduction at \(v\mid 2, 3\), [Halberstadt] over \(\mathbb{Q}\), [Kobayashi] for \(v\mid 3\), [DD-Root2, Whitehouse] for \(v\mid 2\), alternative formula from the proof of parity [DD-Parity]; for abelian varieties see [Sabitova-PhD, Sabitova-Root].

Finiteness of III implies parity:
[Monsky] (over \(\mathbb{Q}\)), [DD-Squarity] (\(E\) semistable at additive \(v\mid 2, 3\)), [DD-Parity] (all cases).

\(p\)-Parity Conjecture proofs:
\(p\)-parity over \(\mathbb{Q}\): [Birch-Stephens], [Greenberg], [Guo], [Monsky], [Nek-Sel], [BDKim], [DD-Squarity];
\(p\)-parity over totally real fields: [Nek-Sel], [Nek-SelIV], [BDKim2], [DD-Parity].

\(p\)-Parity Conjecture for curves with a \(p\)-isogeny:
[Monsky] (over \(\mathbb{Q}\), [DD-Isogeny] (\(E\) semistable at \(p\)) [CFKS] (Abelian varieties semistable at \(p\)), [DD-Parity] (Conj. in all cases + deformation argument for \(p \leq 5\); [TrihWuthr] (function fields).

Brauer relations & compatibility of root numbers and Tamagawa numbers:
Regulator constants [DD-Squarity], their compatibility with Tamagawa numbers [DD-Selfduality], their compatibility with root numbers [DD-Regconst], deformation argument [DD-Parity].

\(p\)-parity for dihedral twists:
[DD-Squarity] or [DD-Regconst] (\(E\) semistable at \(v\mid p\)), [DD-Parity] (\(p \equiv 3 \mod 4\) using deformation argument), [Rochefoucauld] (all \(p \geq 5\) using congruences between root numbers); see also [MR-LocConst, MR-LocConst2] for an alternative approach to the same parity using ‘theory of local constants’ and [DD-Regconst] §4 to applications for rank growth in \(p\)-adic towers.
REFERENCES


