

Background on BSD and its statement over number fields:

[Birch1], [Birch2], [Tate-BSD]; Cassels papers on III [CasselsIV, CasselsIV] (+ its non-squareness for abelian varieties [PoonenStoll]); ζ - and L -functions [Serre-ZetaL]; l -adic representations [Serre-AbLAdic].

Potential modularity & using Brauer/Solomon induction arguments to reduce to modular curves: the course followed [DD-Parity] §2; for background and proofs see [Taylor3] proof of Thm. 2.4, [Taylor4] proof of Cor. 2.2 and Wintenberger's appendix to [Nek-SelIV].

BSD for rank ≤ 1 :

Gross-Zagier formula [GrossZagier] ([Zhang, TianZhang] over totally real fields) + Kolyvagin's Euler system argument [Kolyvagin] + existence of rank ≤ 1 quadratic twist [BFH], [MurtyMurty], [Waldspurger];

Parity predictions:

Twists of semistable curves [Rohrlich-Root, D-Root]; No local expression for the rank [DD-ModN]; Failure of Goldfeld over number fields [Connell, DD-Evil].

Galois action on the Tate module:

[SerreTate], [Serre-PropGal]; the inertia action for $v \nmid 2, 3$ can be recovered from Tate's algorithm [Tate-alg], for $v|2, 3$ see [Kraus, DD-Isogeny].

Weil-Deligne representations and root numbers:

Definitions, existence of ϵ -factors and main properties [DeligneC, Tate-NThBg]; Frohlich-Queyrut theorem [FrohQuey]; Galois invariance of root numbers [Rohrlich-Inv]; see also summary in [DD-Regconst] Appendix A.

Root numbers of elliptic curves:

[Rohrlich-Root] (all cases except additive reduction at $v|2, 3$), [Halberstadt] over \mathbb{Q} , [Kobayashi] for $v|3$, [DD-Root2, Whitehouse] for $v|2$, alternative formula from the proof of parity [DD-Parity]; for abelian varieties see [Sabitova-PhD, Sabitova-Root].

Finiteness of III implies parity:

[Monsky] (over \mathbb{Q}), [DD-Squarity] (E semistable at additive $v|2, 3$), [DD-Parity] (all cases).

p -Parity Conjecture proofs:

p -parity over \mathbb{Q} : [Birch-Stephens], [Greenberg], [Guo], [Monsky], [Nek-Sel], [BDKim], [DD-Squarity]; p -parity over totally real fields: [Nek-Sel], [Nek-SelIV], [BDKim2], [DD-Parity].

p -Parity Conjecture for curves with a p -isogeny:

[Monsky] (over \mathbb{Q}), [DD-Isogeny] (E semistable at p) [CFKS] (Abelian varieties semistable at p), [DD-Parity] (Conj. in all cases + deformation argument for $p \leq 5$); [TrihWuthr] (function fields).

Brauer relations & compatibility of root numbers and Tamagawa numbers:

Regulator constants [DD-Squarity], their compatibility with Tamagawa numbers [DD-Selfduality], their compatibility with root numbers [DD-Regconst], deformation argument [DD-Parity].

p -parity for dihedral twists:

[DD-Squarity] or [DD-Regconst] (E semistable at $v|p$), [DD-Parity] ($p \equiv 3 \pmod{4}$ using deformation argument), [RocheFoucauld] (all $p \geq 5$ using congruences between root numbers); see also [MR-LocConst, MR-LocConst2] for an alternative approach to the same parity using 'theory of local constants' and [DD-Regconst] §4 to applications for rank growth in p -adic towers.

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