MECH1 **Problem Sheet 5** **Due Monday April 27**

Work and Energy, continued

Do all problems

1. (This is the last problem from the previous homework) The one-dimensional potential energy function for a particle is given as $U(x) = \frac{1}{x^3} - \frac{1}{x}$. Sketch U, identify and describe equilibrium points and regions of finite/infinite motion.

2. Given $\boldsymbol{F}(x, y) = xy\boldsymbol{i} + \frac{y}{x}\boldsymbol{j}$ compute the work of $\boldsymbol{F}$ along the path $y = x^2$, between the origin and the point $(1, 1)$. Is this a potential force?

3. A particle is moving in three dimensions along the right helix of radius R and pitch (the vertical distance between nearby loops) H. Given $\boldsymbol{F}(x, y, z) = y\boldsymbol{i} + x\boldsymbol{j} + xyz\boldsymbol{k}$ compute the work of $\boldsymbol{F}$ along the part of the helix that starts at $(R, 0, 0)$ and ends at $(-R, 0, H/2)$. (Help: the helix gets parameterised as $x = R\cos t,\ y = R\sin t,\ z = \frac{H}{2\pi}t$. Why?) Is this a potential force?

4. Suppose a force $\boldsymbol{F}(\boldsymbol{r})$ is central. I.e. $\boldsymbol{F}(\boldsymbol{r}) = f(r)\frac{\boldsymbol{r}}{r}$, where r is the distance form the origin. Show that such a force is potential (i) by definition of work, showing that the integral involved between any points A and B depends only on the distances $r(A)$ and $r(B)$ from the origin, but not on the particular path connecting A and B; (ii) by demonstrating that, in fact, if a function of one variable $U(r)$ is such that $U'(r) = -f(r)$, then $\boldsymbol{F}(\boldsymbol{r}) = -\nabla U$.

   Then find the potential corresponding to the gravitational interaction force

$$\boldsymbol{F}(\boldsymbol{r}) = G\frac{m_1 m_2}{r^2}\frac{\boldsymbol{r}}{r}.$$

5. A particle of mass m_1 moves with constant velocity v_1 and hits the resting particle of mass m_2. After that the two particles stick together and continue to move as a single particle of mass $m_1 + m_2$. Use conservation of momentum to show that the collision resulted in the loss of kinetic energy. Calculate this loss. Discuss where the mechanical energy could go.

6. A ballistic pendulum is a heavy sack filled with sand of mass M, suspended on a string of length l. A bullet of mass m traveling horizontally hits the sack and gets stuck in it, whereupon the sack starts oscillating with angular amplitude α. Find the velocity of the bullet before it hits the sack.