MECH1 **Problem Sheet 6** **Due pm on Tuesday May 5**

Constants of motion, angular momentum

1. The net force on a particle moving in three dimensions is $\boldsymbol{F} = a\boldsymbol{i} + b\boldsymbol{j}$ with $a, b \neq 0$. Find the direction(s) in which (i) the momentum, (ii) angular momentum are conserved.

2. A bead of mass m whirls on the frictionless table, held to circular motion by a string that passes through a hole in the centre of the table. The string is *slowly* pulled through the hole, so that at every moment t the bead can be thought to be in the state of circular motion, with radius r and angular velocity $\omega(r)$. Originally $r = r_0$ and $\omega = \omega_0$.

   Sow that $\omega(r)r^2 = const.$. Use this to express the bead's kinetic energy on r. Verify that as the distance from the origin decreases from r_0 to r, the change of kinetic energy equals the work of the force $T(r)$ impelled by the rope. HINT: How does *slowly* enable one to express the latter force as a function of r?

Central force field, polar coordinates

1. Can a repelling central force result in circular orbits?

2. A particle moves under a cental force $\boldsymbol{F}(\boldsymbol{r}) = -Kr^4\hat{\boldsymbol{r}}$, with $K > 0$ and the notation $\hat{\boldsymbol{r}} = \frac{\boldsymbol{r}}{r}$ for the unit vector in the direction of $\boldsymbol{r}$. Is this an attractive or repelling force? Given the angular momentum value L, find the radius, energy and period of the circular orbit in terms of L.

   Eliminate L and find the angular velocity $\dot{\theta}$ on the circular orbit as a function of its radius R.

   Suppose at time $t = 0$ one has $\dot{r}(0) \neq 0$. Is such an initial condition compatible with moving on a circular orbit.

3. A particle of mass m moves in the central force field with the force function $f(r) = -Kr^3$, with $K > 0$. Sketch the effective potential, and argue that all the orbits are bounded. Find the radius and period of circular orbit(s), if any.

4. A particle of mass m moves in the central force field with the force function $f(r) = -K/r^3$, with $K > 0$. Find the nonzero values of the angular momentum L when circular orbits are possible. For these particular values of L, describe the (non-circular) orbits, given that initially the particle's velocity had some non-zero radial component.