MECH1 **Problem Sheet 7** **Due Monday May 11**

Oscillations

Do all problems. Terminology: the damped harmonic oscillator equation is

$$\ddot{x} + 2\gamma\dot{x} + \omega^2 x = 0,$$

if the right-hand side is not zero, it is referred to as forcing. Always assume that $\omega \gg \gamma$.

1. (This is a revision-type problem: consider it in the mass centre frame.) Find a period of oscillations of a system that consists of two masses m and M, connected by a massless spring with constant k. Answer: $2\pi\sqrt{\frac{\mu}{k}}$, with $\mu = \frac{mM}{m+M}$.

2. A mass m can slide without friction on a horizontal plane, being attached to a vertical wall via two consecutive springs with constants k and K. Find the period of oscillations. Answer: $2\pi\sqrt{\frac{m}{\kappa}}$, with $\kappa = \frac{kK}{k+K}$. (This is also a revision-type problem:argue that the net force acting on the point where the springs are connected must be zero.)

   Describe what would change in your solution if the mass was hanging vertically.

3. Show that the solution of
   $$\ddot{x} + \omega^2 x = f\cos{(\omega + \epsilon)t}$$
   with $x = \dot{x} = 0$ can be written as
   $$x = \frac{2f}{\epsilon^2 + 2\omega\epsilon}\sin\frac{1}{2}\epsilon t\,\sin(\omega + \frac{1}{2}\epsilon)t.$$
   (Use the formula $\cos u - \cos v = -2\sin\frac{u+v}{2}\sin\frac{u-v}{2}$.)

   Sketch qualitatively this solution for a small ϵ.

   Discuss what happens to it when ϵ goes to zero when resonance is achieved.

4. A damped harmonic oscillator with constants γ, ω is forced by a sine force with the frequency Ω. Find the value of Ω where the steady state solution has maximum amplitude.

   Suppose, at the time $t = 1000$, after forcing has been on for a long time, the solution $x(t)$ passes through a local maximum. At this very moment the external forcing gets suddenly switched off. Sketch $x(t)$ qualitatively before and after this has happened.