Quantifying connectivity of ad-hoc networks

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We have discovered universal properties of confined random geometric graphs by developing mathematical models for ad-hoc networks. We use these models to design reliable wireless mesh networks at reduced deployment and running costs.

Urbanisation is a significant worldwide trend which Smart City technologies aim to address. These rely heavily on wireless communications for sensing and control purposes; however the cost and complexity of planning and deploying such infrastructures is often prohibitive, a problem that this research aims to address.

1 A collection of nodes randomly distributed in some finite domain, pairwise connected with a probability depending on mutual distances.
2 Networks which do not rely on a pre-existing infrastructure and can be deployed "on the fly".

Universality in network connectivity

Using (1) and (2), we have derived a general expression for network connectivity in the high density regime:

\[ P_{fc} \approx 1 - \rho \sum_{B} G_B V_{B} e^{-\rho M_B} \]

- A sum of separable boundary contributions \( B \) which exhibit universal properties, distinct but complementary to those of classical percolation phenomena.

- Arbitrary convex geometries in any dimension.
- Independent of the connectivity model \( H_g \).
- First uniform treatment of boundary effects across density regimes.

"Connectivity is governed by the microscopic details of the network domain such as sharp corners rather than the macroscopic total volume".

Network reliability

This can be quantified by k-connectivity; the property that the network remains connected if any k-1 nodes fail. Our analytic expressions provide a useful tool for design specifications.

Anisotropic radiation patterns

Ad-hoc networks with randomly oriented directional antenna gains \( G(\theta) \) have fewer short links and more long links which can bridge otherwise isolated sub-networks. Whether this is advantageous (or not) is governed by the functional:

\[ S_B[G] = \int_{0}^{\pi} \sin \theta G(\theta)^{3/4} d\theta \]


Non-convex domains

The network is connected if its sub-networks are connected, and there exists at least one "bridging" link X through the opening:

\[ P_{f_c}^{(V)} = P_{f_c}^{(A)} P_{f_c}^{(B)} X \]

Semi-quenched disorder

To analyse the bridging link we keep nodes on one side frozen ("quenched") while averaging over the positions of those on the other side. This realization leads to:

\[ X = 1 - \left( \prod_{i=1}^{N_A} \prod_{j=1}^{N_B} (1 - \chi_{ij} H_{ij}) b_{ij} \right) A \]

For a Rayleigh fading model: \( H_{ij} = e^{-\frac{d_{ij}}{2r_{0}}^2} \) we can calculate:

\[ X \approx 1 - \exp \left( -\rho A b_{ij} 2r_{0}^2 \right) \]

A step closer to Smart-Cities

By quantifying the connectivity of ad-hoc networks, we can control key features such as robustness to failure and identify the most cost effective methods for optimal deployment.