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"designing mesh networks, easily"

Stochastic geometry, Network theory, Statistical physics.

Quantifying connectivity of ad-hoc networks

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We have discovered universal properties of *confined random geometric graphs*[1] by developing mathematical models for *ad-hoc networks*[2]. We use these models to design reliable wireless mesh networks at reduced deployment and running costs.

Urbanisation is a significant worldwide trend which *Smart City* technologies aim to address. These rely heavily on wireless communications for sensing and control purposes; however the cost and complexity of planning and deploying such infrastructures is often prohibitive, a problem that this research aims to address.

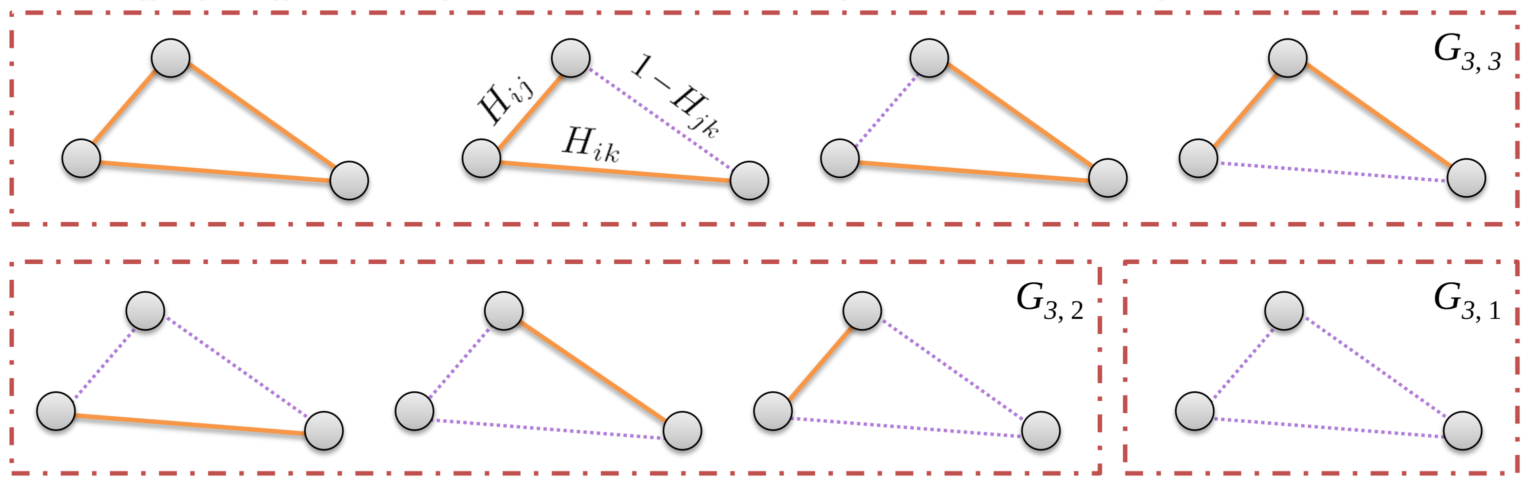
[1] A collection of nodes randomly distributed in some finite domain, pairwise connected with a probability depending on mutual distances.

[2] Networks which do not rely on a pre-existing infrastructure and can be deployed "on the fly".

Universality in network connectivity

Model

Group graphs together according to their largest cluster size:



$$1 = \underbrace{\sum_{g\in G_{N,N}} \mathcal{H}_g}_{P_{fc}} + \sum_{g\in G_{N,N-1}} \mathcal{H}_g + \ldots + \underbrace{\sum_{g\in G_{N,1}} \mathcal{H}_g}_{\prod_{i<j}(1-H_{ij})}$$

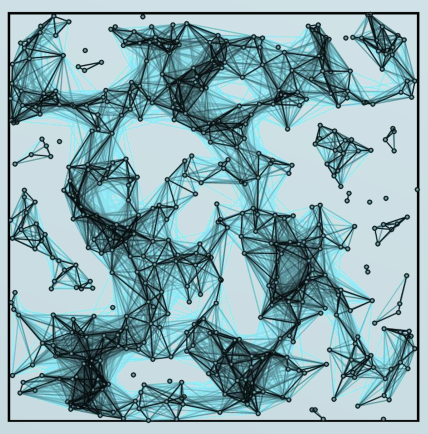
Rearranging we can study the probability of *full connectivity*:

$$P_{fc} = 1 - \sum_{g\in G_{N,N-1}} \mathcal{H}_g - \ldots \qquad (1)$$

Expectation values are averages over spatial realizations:

$$\langle A\rangle = \frac{1}{V^N}\int_{\mathcal{V}^N} A(\mathbf{r}_1,\ldots,\mathbf{r}_N)\mathrm{d}\mathbf{r}_1\ldots\mathrm{d}\mathbf{r}_N \qquad (2)$$

Symbol	Definition / Explanation
$H_{ij} = H(r_{ij})$	Probability that nodes *i* and *j* connect
$\mathcal{V}$ / V	Network domain / Volume
N	Number of nodes
$\rho = N/V$	Density of nodes
$\mathcal{H}_g$	Probability of graph *g*
$G_{N,S}$	Set of graphs with largest cluster of size *S*



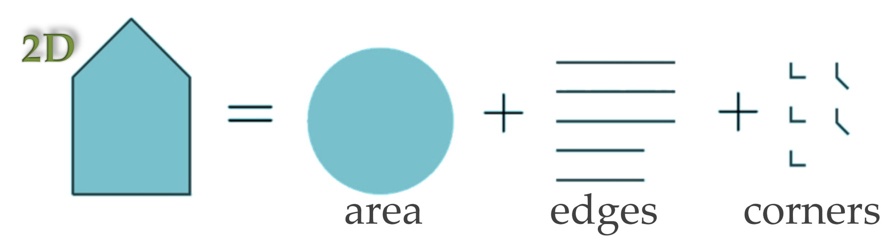
Network Clustering

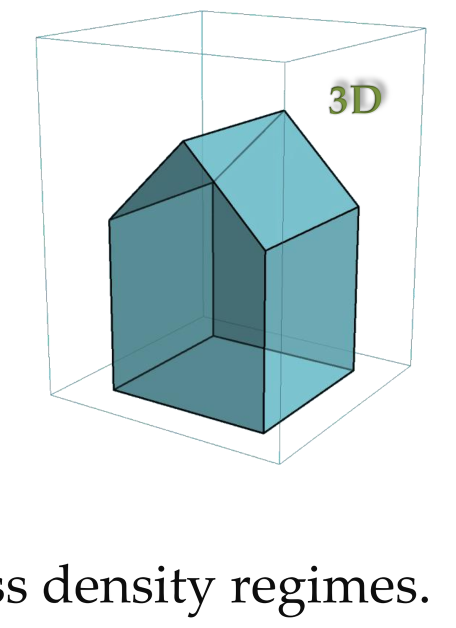
Results

Using (1) and (2), we have derived a general expression for network connectivity in the high density regime:

$$P_{fc} \approx 1 - \rho \sum_B G_B V_B e^{-\rho M_B}$$

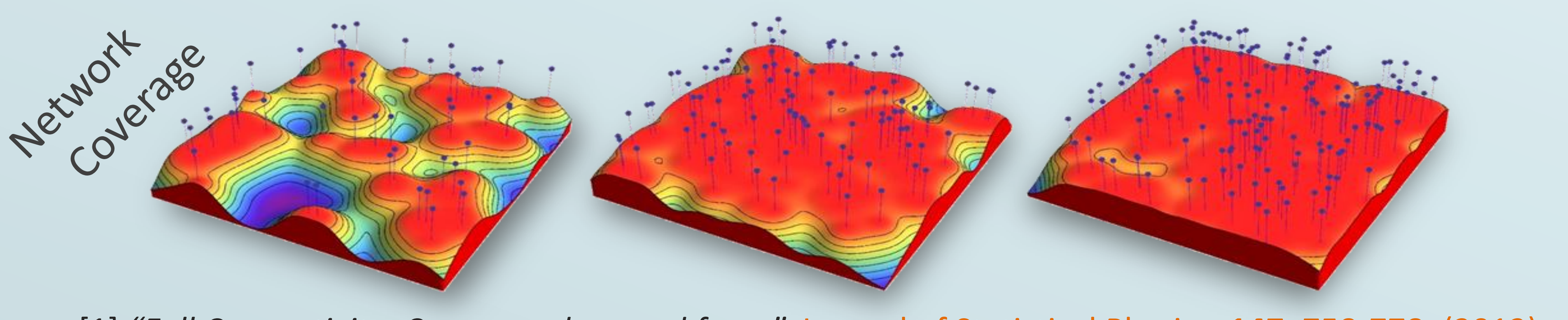
- A sum of separable boundary contributions *B* which exhibit universal properties, distinct but complementary to those of classical percolation phenomena.





- Arbitrary convex geometries in any dimension.
- Independent of the connectivity model H_{ij}.
- First uniform treatment of boundary effects across density regimes.

"Connectivity is governed by the microscopic details of the network domain such as sharp corners rather than the macroscopic total volume".

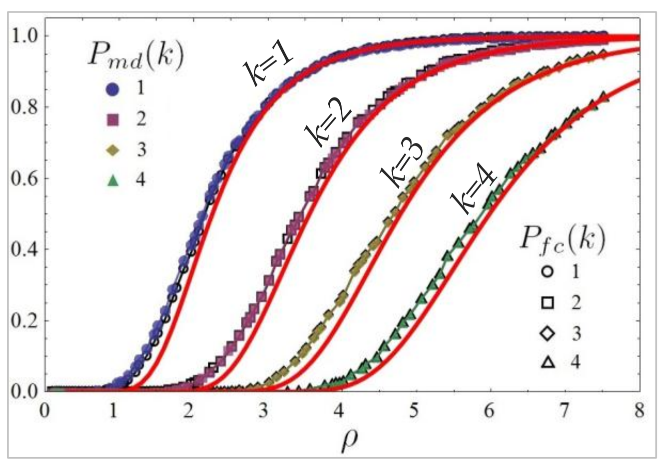


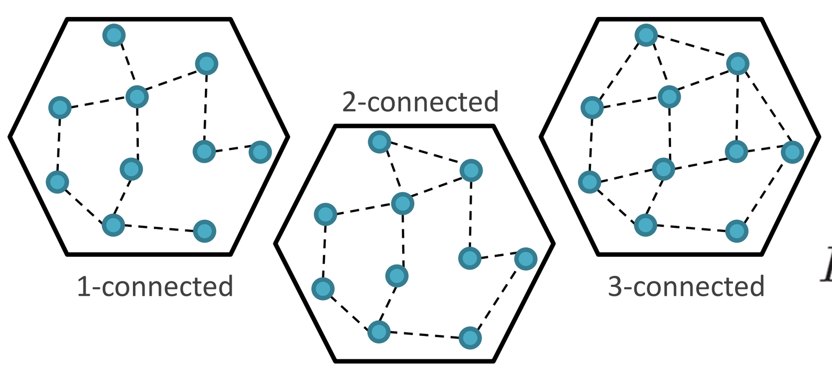
[1] *"Full Connectivity: Corners, edges and faces"*, Journal of Statistical Physics, **147**, 758-778, (2012).

[2] *"Impact of boundaries on fully connected random geometric networks"*, PRE, **85**, 011138, (2012).

Network reliability

This can be quantified by *k*-connectivity; the property that the network remains connected if any *k-1* nodes fail. Our analytic expressions provide a useful tool for design specifications.





$$M(\mathbf{r}_i) = \int_{\mathcal{V}} H(r_{ij})\mathrm{d}\mathbf{r}_j$$

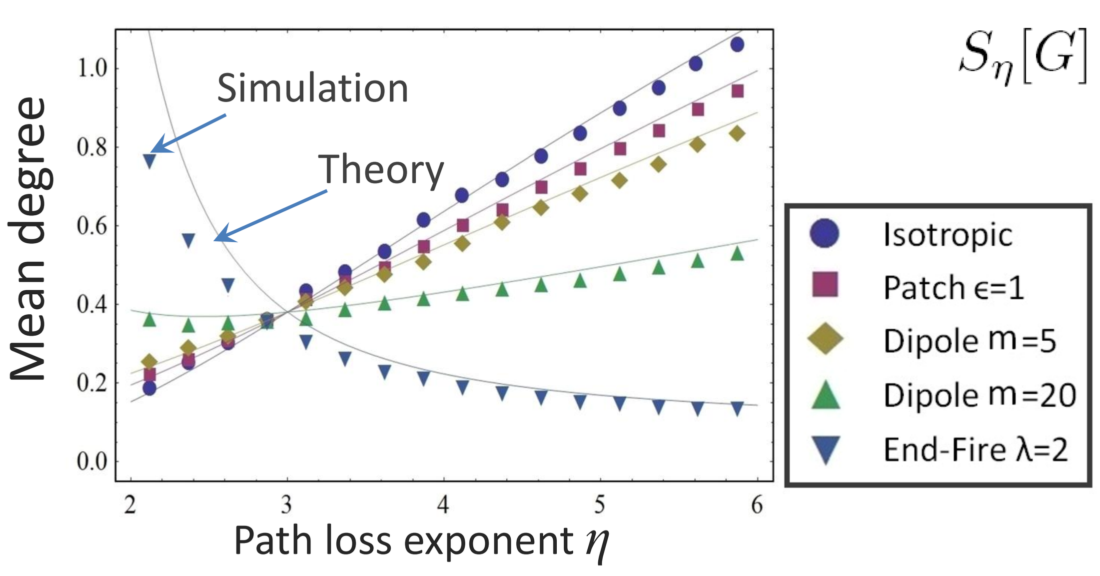
$$P_{fc}(k) \approx 1 - \sum_{m=0}^{k-1} \frac{\rho^{m+1}}{m!}\int_{\mathcal{V}} M(\mathbf{r}_i)^m e^{-\rho M(\mathbf{r}_i)}\mathrm{d}\mathbf{r}_i$$

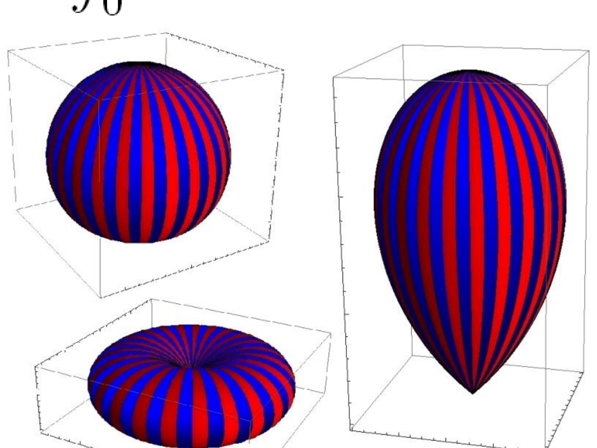
[3] *"k-connectivity for confined random networks"*, Europhysics Letters, **103**, 28006, (2013).

Anisotropic radiation patterns

Ad-hoc networks with randomly oriented *directional* antenna gains $G(\theta)$ have fewer short links and more long links which can bridge together otherwise isolated sub-networks. Whether this is advantageous (or not) is governed by the functional:

$$S_\eta[G] = \int_0^\pi \sin\theta G(\theta)^{3/\eta}\mathrm{d}\theta$$





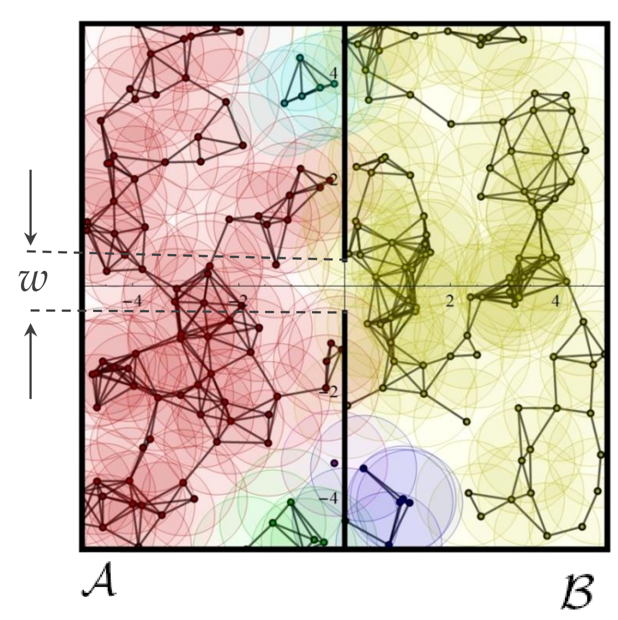
[5] *"Connectivity of confined 3D networks with anisotropically radiating nodes"*, arXiv:1310.7473, (2014).

Non-convex domains

The network is connected if its sub-networks are connected, and there exists at least one *"bridging"* link *X* through the opening:

$$P_{fc}^{(\mathcal{V})} = P_{fc}^{(\mathcal{A})} P_{fc}^{(\mathcal{B})} X$$

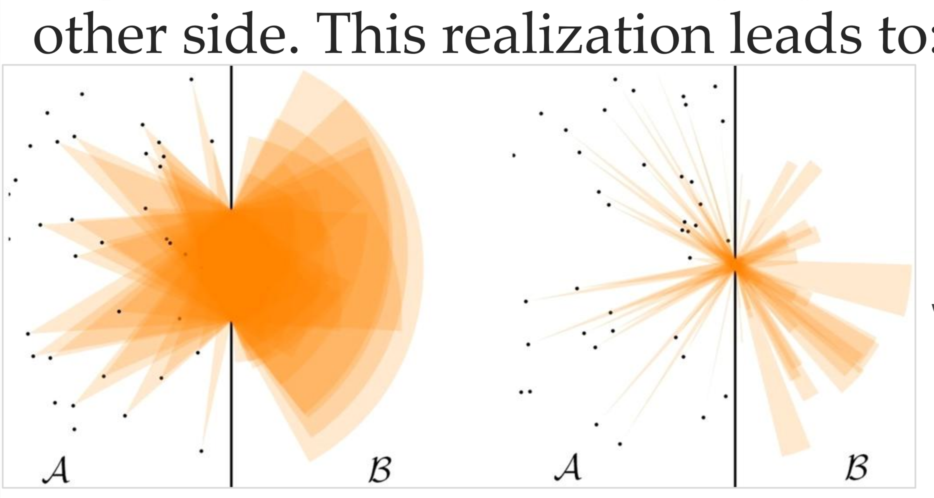
$$X = 1 - \langle\langle \prod_{i=1}^{N_{\mathcal{A}}} \prod_{j=1}^{N_{\mathcal{B}}} (1-\chi_{ij}H_{ij})\rangle_{\mathcal{B}}\rangle_{\mathcal{A}}$$



Semi-quenched disorder

To analyse the bridging link we keep nodes on one side frozen ("quenched") while averaging over the positions of those on the other side. This realization leads to:

$$X = 1 - \langle \prod_{i=1}^{N_{\mathcal{A}}} \langle \prod_{j=1}^{N_{\mathcal{B}}} (1-\chi_{ij}H_{ij})\rangle_{\mathcal{B}}\rangle_{\mathcal{A}}$$



For a Rayleigh fading model: $H_{ij} = e^{-\left(\frac{r_{ij}}{r_0}\right)^\eta}$ we can calculate :

$$X \approx 1 - \exp\left(-\rho_{\mathcal{A}}\rho_{\mathcal{B}}\frac{2r_0^3 w}{3}\right)$$

[4] *"Network connectivity through small openings"*, Best Paper in proceedings ISWCS'13, (2013).

A step closer to *Smart-Cities*

By quantifying the connectivity of *ad-hoc* networks, we can control key features such as robustness to failure and identify the most cost effective methods for optimal deployment.

http://maps.bristol.gov.uk/pinpoint