Suppose that \( \{ \theta_i, i = 1, 2, \ldots, n \} \) are random variables in \( \Omega \), that \( G_0 \) is an arbitrary distribution on \( \Omega \), \( \alpha > 0 \) is a positive real, and that given \( \{ \theta_i \} \), \( \{ Y_i, i = 1, 2, \ldots, n \} \) are independent, with \( Y_i | \theta_i \sim f(\cdot | \theta_i) \). The following models are equivalent (in all cases the \( \theta_i \) are exchangeable):

**Ferguson definition of Dirichlet process**

The distribution \( G \) on \( \Omega \) is drawn from the Dirichlet process DP(\( \alpha, G_0 \)), i.e. for all partitions \( \Omega = \bigcup_{j=1}^m B_j \) (\( B_j \cap B_k = \emptyset \) if \( j \neq k \)), and for all \( m \),

\[
(G(B_1), \ldots, G(B_m)) \sim \text{Dirichlet}(\alpha G_0(B_1), \ldots, \alpha G_0(B_m))
\]

Then, given \( G \), \( \{ \theta_i \} \) are drawn i.i.d. from \( G \).

**Stick-breaking**

\( G \) is constructed as \( \sum_{j=1}^{\infty} w_j \delta_{\theta^*_j} \) where \( w_j = \prod_{r=1}^{j-1} (1 - V_r) V_j, V_r \sim \text{Beta}(1, \alpha) \) are i.i.d. and \( \theta^*_j \sim G_0 \) are i.i.d. and independent of \( \{ V_r \} \). Then given \( G \), \( \{ \theta_i \} \) are again drawn i.i.d. from \( G \).

**Limit of finite mixtures**

Draw the \( Y_i \) i.i.d. from the finite mixture model \( \sum_{j=1}^{k} w_j f(\cdot | \theta^*_j) \) where \( (w_1, w_2, \ldots, w_k) \sim \text{Dirichlet}(\alpha/k, \alpha/k, \ldots, \alpha/k) \) and \( \theta^*_j \sim G_0 \) are i.i.d. and independent of \( \{ w_j \} \). Then let \( k \to \infty \).

**A partition model**

Partition \( \{ 1, 2, \ldots, n \} = \bigcup_{j=1}^{d} C_j \) at random so that \( p(C_1, C_2, \ldots, C_d) = (\Gamma(\alpha)/\Gamma(\alpha + n)) \)

\[
\alpha^d \prod_{j=1}^{d} (n_j - 1)! \text{ where } n_j = \#C_j.
\]

Draw \( \theta^*_j \sim G_0 \) i.i.d. for \( j = 1, 2, \ldots, d \). For all \( i \in C_j \), set \( \theta_i = \theta^*_j \).

**Pólya urn representation**

Draw \( \theta_1 \sim G_0 \), and then for all \( i = 1, 2, \ldots, n - 1 \), draw \( \theta_{i+1} | \theta_1, \theta_2, \ldots, \theta_i \sim (\alpha/(\alpha + i))G_0 + (1/(\alpha + i)) \sum_{r=1}^{i} \delta_{\theta_r} \).

**Species sampling model (Pólya urn representation using allocations)**

Set \( z_1 = 1 \), then for all \( i = 1, 2, \ldots, n - 1 \), draw \( z_{i+1} | z_1, z_2, \ldots, z_i \sim (\alpha/(\alpha + i))\delta_{d_i} + (1/(\alpha + i)) \sum_{r=1}^{i} \delta_{z_r} \) where \( d_i = \max\{ z_1, z_2, \ldots, z_i \} \). Then draw \( \theta^*_j \sim G_0 \) i.i.d., and set \( \theta_i = \theta^*_z \).