# MATH11007 SHEET 20: LAGRANGE MULTIPLIERS, DIFFERENTIAL FORMS

(1) Use the method of Lagrange multipliers to find the stationary points of the function $f(x, y) = x^2 - y^2$ subject to the constraint $x^2 + y^2 = 1$. Identify for each point whether it gives a maximum or or minimum.

(2) Use the method of Lagrange multipliers to find the minimum value of the function $f(x, y, z) = x^2 + 2y^2 + 3z^2$ subject to the two constraints $x + y = 1$ and $y + z = 1$.

(3) A rectangular box without a lid is to be made from exactly $12\text{m}^2$ of cardboard. Use the method of Lagrange multipliers to find the maximum volume of such a box.

(4) Using the properties of the wedge product, show that
   (a) $dx \wedge dy \wedge dz = dz \wedge dx \wedge dy = dy \wedge dz \wedge dx$
   (b) $(3dx + dy) \wedge (dx + 2dy) = 5dx \wedge dy$.

(5) (a) For the 1-form $\omega = F(x, y, z)dx + G(x, y, z)dy + H(x, y, z)dz$ show that $d\omega = (G_x - F_y)dx \wedge dy + (H_y - G_z)dy \wedge dz + (F_z - H_x)dz \wedge dx$.
   (b) For the 2-form $\alpha = F(x, y, z)dy \wedge dz + G(x, y, z)dz \wedge dx + H(x, y, z)dx \wedge dy$ show that $d\alpha = (F_x + G_y + H_z)dx \wedge dy \wedge dz$.

(6) A fundamental property of the exterior derivative is that $d^2 = 0$. In this problem you are asked to verify this explicitly for functions (0 forms) and 1 forms.
   (a) For a function $f$ show that $d(df) = d^2 f = 0$.
   (b) For a 1-form $\omega = F(x, y, z)dx + G(x, y, z)dy + H(x, y, z)dz$ show that $d(d\omega) = d^2\omega = 0$, where the coefficents must be sufficiently differentiable so that the "mixed partial derivatives" are equal ( $C^2$ is sufficient).

## References


1. *Lagrange multipliers material:*
   `http://www.maths.bris.ac.uk/~mayt/MATH11007/2010/notes/LagrangeMultiplierHandout.pdf`
   `http://www.maths.bris.ac.uk/~mayt/MATH11007/2010/notes/LagrangeMultiplierLecture.pdf`
2. *Differential forms material:*
   `http://www.maths.bris.ac.uk/~mayt/MATH11007/2010/notes/DifferentialFormsLecture.pdf`
3. Donu Arapura, *Introduction to Differential Forms*,
   `http://www.maths.bris.ac.uk/~mayt/MATH11007/2010/notes/arapura.pdf`