

MATH11007 SHEET 18: THE DIRECTIONAL DERIVATIVE.

Set on Tuesday, March 6: Qs 1, 2 and 5.

- (1) In what direction(s) does the directional derivative of $f(x, y) = xy$ at the point $(2, 0)$ take the value -1 ?
- (2) You are given that the directional derivative of a function $f(x, y)$ at a point (a, b) in the direction of the vector $(1, 2)$ is 2, and in the direction of the vector $(-1, 1)$ is 3. Find the partial derivatives of f at (a, b) .
- (3) Find the directional derivative of $f(x, y) = \sqrt{xy}$ at the point $P(2, 8)$ in the direction of the point $Q(5, 4)$.
- (4) Find the maximum rate of change of $f(x, y, z) = \ln |xy^2z^3|$ at the point $(1, -2, -3)$. In what direction does it occur?
- (5) For each of the following, find a normal vector to the given curve (or surface) at the given point, and the equation of the tangent line (or plane) at that point:
 - (a) $x^2y^2 = 1$ at $(2, -1/2)$.
 - (b) $x^2 + 2y^2 + 3z^2 = 6$ at $(1, -1, 1)$.
 - (c) $z = p^4 - x^2 - y^2$ at $(1, -1, p^2)$.
- (6) You are standing on a very steep hillside; the height of the ground at a point (x, y) is given by $f(x, y)$. If you head due North, you go uphill at a slope of 60° ; if you head due East, you go uphill at 30° . In which direction is the slope greatest? What is the value of the greatest slope?
- (7) Let $z = f(x, y) = x^2y$, and let $\mathbf{u} = (u, v)$ be a unit vector. Write a formula for $D_{\mathbf{u}}f(1, -1)$ in terms of u and v . Find u and v such that the derivative is zero.

REFERENCES

1. Frank Ayres, Jr. and Elliott Mendelson, *Schaum's Outline of Calculus, Fourth Edition* Chapters 48 and 52, Mc-Graw-Hill, 1999.