MATH11007 SHEET 20: INTEGRATION OVER TWO-DIMENSIONAL DOMAINS

Set on Monday, March 19: Qs 2 and 3.

(1) Evaluate

$$\text{(a)}\ \frac{\mathrm{d}}{\mathrm{d}s}\int_0^s \mathrm{e}^{st^2}\,\mathrm{d}t\,; \quad \text{(b)}\ \frac{\mathrm{d}}{\mathrm{d}a}\int_a^{a^2} \sin(ax+x^2)\,\mathrm{d}x\,.$$

(2) Evaluate

$$\text{(a)}\ \int_2^6\int_0^2 (x-y)\,\mathrm{d}y\mathrm{d}x\,; \quad \text{(b)}\ \int_0^{\pi/2}\int_0^{\pi/4} \sin(x+y)\,\mathrm{d}x\mathrm{d}y\,.$$

(3) In each case, sketch the region of integration, and write down the integral with the order of integration reversed.

(a)
$$\int_0^2\int_{x^2}^{2x} f(x,y)\,\mathrm{d}y\mathrm{d}x$$

(b)
$$\int_1^2\int_0^{\ln x} f(x,y)\,\mathrm{d}y\mathrm{d}x$$

(c)
$$\int_0^1\int_{1-y}^{\sqrt{1-y^2}} \phi(x,y)\,\mathrm{d}x\mathrm{d}y\,.$$

(4) Evaluate

$$\text{(a)}\ \int_0^2\int_0^{4-2x} (x+y)\,\mathrm{d}y\mathrm{d}x\,; \quad \text{(b)}\ \int_0^1\int_0^{\sqrt{1-y^2}} y\,\mathrm{d}x\mathrm{d}y\,.$$

(5) Evaluate the following integrals, reversing the order of integration if necessary.

(a)
$$\int_0^{\pi/4}\int_0^{\tan x} \sec x\,\mathrm{d}y\mathrm{d}x\,.$$

(b)
$$\int_0^1\int_0^s t\sqrt{t^2+s^2}\,\mathrm{d}t\mathrm{d}s\,.$$

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(c)

$$\int_0^1 \int_x^1 \cos\left(\xi^2\right) \, \mathrm{d}\xi \mathrm{d}x \, .$$

(6) Find the area of the portion of the plane $x + y + z = 6$ inside the cylinder $x^2 + y^2 = 4$.

References

1. Frank Ayres, Jr. and Elliott Mendelson, *Schaum's Outline of Calculus, Fourth Edition* Chapters 54 and 56, Mc-Graw–Hill, 1999.