MATH11007 SHEET 2: LIMITS

Abstract. The following are slight modifications of questions taken from [1], Chapter 7.

Set on Friday, October 19: Questions 1 and 4.

(1) Evaluate the following limits, if they exist:

(a) $\lim_{x\to 1}(x^2-4x)$; (b) $\lim_{x\to -2}(x^3+2x^2-3x-4)$; (c) $\lim_{x\to 1}\dfrac{(3x-2)^2}{(x+1)^3}$;

(d) $\lim_{x\to 0}\dfrac{\mathrm{e}^x-\mathrm{e}^{-x}}{\mathrm{e}^x+\mathrm{e}^{-x}}$; (e) $\lim_{x\to 3}\dfrac{x-1}{x^2-1}$; (f) $\lim_{x\to 2}\dfrac{x^2-4}{x^2-6x+6}$;

(g) $\lim_{x\to -1}\dfrac{x^2+5x+4}{x^2+4x+3}$; (h) $\lim_{x\to 3}\dfrac{x-3}{x^2-9}$; (i) $\lim_{x\to 3+}\dfrac{x-3}{\sqrt{x^2-9}}$;

(j) $\lim_{x\to 3+}\dfrac{\sqrt{x-3}}{x^2-9}$; (k) $\lim_{h\to 0}\dfrac{(x+h)^4-x^4}{h}$; (l) $\lim_{x\to 2}\dfrac{x-2}{\sqrt{x^2+5}-3}$.

(2) Evaluate the following limits, if they exist:

(a) $\lim_{x\to\infty}\dfrac{8x^9-4x^5+2x-13}{-3x^9+x^8-5x^2+2x}$; (b) $\lim_{x\to 0}\dfrac{8x^9-4x^5+2x-13}{-3x^9+x^8-5x^2+2x}$;

(c) $\lim_{x\to\infty}\dfrac{15x^3-5x+27}{x^3+10}$; (d) $\lim_{x\to -\infty}\dfrac{x^5+x+5}{x^3+6}$;

(e) $\lim_{x\to\infty}(3x^4-25x^3-17)$; (f) $\lim_{x\to -\infty}(-4x+6)^3$.

(3) Evaluate the following limits, if they exist:

(a) $\lim_{x\to\infty}\dfrac{\sqrt{x+3}}{2x-5}$; (b) $\lim_{x\to\infty}\dfrac{2\sqrt{x^2+2}}{6+x}$; (c) $\lim_{x\to\infty}\dfrac{x}{x^2+4}$;

(d) $\lim_{x\to\infty}\dfrac{\mathrm{e}^x-\mathrm{e}^{-x}}{\mathrm{e}^x+\mathrm{e}^{-x}}$; (e) $\lim_{x\to -\infty}\dfrac{\mathrm{e}^x-\mathrm{e}^{-x}}{\mathrm{e}^x+\mathrm{e}^{-x}}$; (f) $\lim_{x\to\infty}\cos x$;

(g) $\lim_{x\to\infty}\dfrac{\cos x}{x}$; (h) $\lim_{x\to\infty}\dfrac{x}{\sin x}$; (i) $\lim_{x\to\infty}\dfrac{\sin x}{\sqrt{x}}$ (j) $\lim_{x\to 0}\dfrac{\sin x}{\sqrt{x}}$.

(4) Find

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

where

(a) $f(x)=\mathrm{e}^x$, $a=2$.

(b) $f(x)=\cos x$, $a\in\mathbb{R}$.

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(c) $f(x) = \sin x$, $a \in \mathbb{R}$.
(d) $f(x) = \sqrt{x}$, $a = 1$.
(e) $f(x) = |x|$, $a = 1$.
(f) $f(x) = |x|$, $a = -1$.
(g) $f(x) = \tan x$, $a \in (-\pi/2, \pi/2)$.
(h) $f(x) = \sin^2 x$, $a \in \mathbb{R}$.

(5) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x+1 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}.$$

(a) Draw the graph of this function.
(b) Find the limits

$$\lim_{x \to 0-} f(x) \quad \text{and} \quad \lim_{x \to 0+} f(x)\,.$$

(c) Is this function continuous?

References

1. Frank Ayres, Jr. and Elliott Mendelson, *Schaum's Outline of Calculus, Fourth Edition*, McGraw–Hill, 1999.