# HW4, Bayesian Modelling B 2016/17

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In the homeworks, questions with marks are officially ‘exam-style’, although you can expect any homework question to appear as an exam question, unless it is explicitly ‘not examinable’.

Hand in Q2, Q3, and Q6.

1. Prove that convergence in mean square implies convergence in probability. [10 marks]

2. State Brouwer’s Fixed Point Theorem, and prove it, using a diagram, in the 1D case. Prove that every transition matrix $P$ has at least one fixed point. [10 marks]

3. Suppose that $X_0, X_1, \dots$ are IID. What does the transition matrix look like? What is the stationary distribution? Is it unique? [10 marks]

4. Let $P$ and $Q$ be transition matrices with a common stationary distribution $\pi$.

   (a) Prove that if $0 \leq \alpha \leq 1$, then $R = \alpha P + (1 - \alpha) Q$ also has stationary distribution $\pi$. Explain how sampling from $R$ can be implemented in practice. [10 marks]

   (b) Prove that $R = PQ$ also has stationary distribution $\pi$. Explain how sampling from $R$ can be implemented in practice. [10 marks]

5. Suppose that a transition matrix $P$ can be written in terms of two smaller transition matrices $P_1$ and $P_2$ as
$$P = \begin{pmatrix} P_1 & \mathbf{0} \\ \mathbf{0} & P_2 \end{pmatrix}.$$
Show that $P$ is not irreducible, and that it has multiple stationary distributions. [10 marks]

6. Consider the transition matrix
$$P = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$
How would you demonstrate that this matrix is irreducible and aperiodic? How could you approximate its unique stationary distribution using R? [10 marks]