Uncertainty and Risk in Natural Hazards

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Natural hazards
A general framework for natural hazards

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2. The *hazard process* defined on the domain. This comprises a set \( \Omega \) of mutually-exclusive *hazard outcomes*, and a set of probabilities, \( P = \{p_\omega : \omega \in \Omega\} \). These probabilities represent *aleatory uncertainty*. 
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Not knowing these components, or limitations in doing the calculations, leads to epistemic uncertainty.
A concrete example: Earthquake!!

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5. The loss might then be the total area of damage in the region, with respect to some threshold, say 1 m/s²,

\[
l_\omega = \sum_{x \in \mathcal{X}} \text{area}(x) 1[ v_\omega(x) > 1 ].
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Treating $\omega$ as uncertain, $\ell_\omega$ becomes an uncertain quantity which we label as $\tilde{\ell}$. The distribution of $\tilde{\ell}$ is a summary of aleatory earthquake uncertainty in the domain, in terms of the impact on structures.
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The probability distribution function of $\tilde{\ell}$ is

$$F_{\tilde{\ell}}(v) = \Pr\{\tilde{\ell} \leq v\} = \sum_{\omega \in \Omega} 1[\ell_\omega \leq v]p_\omega.$$  

The PE curve is the plot of $v$ (x-axis) against $1 - F_{\tilde{\ell}}(v)$. 

![Probability of exceedence (PE) curve](image-url)
Probability of exceedence (PE) curves

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![Graph showing probability of exceedence (PE) curves][1]
Accounting for *epistemic uncertainty*

The current state of the art . . .
Accounting for **epistemic uncertainty**

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A catalogue of epistemic uncertainties

In order of increasing challenge:

1. Not being able to enumerate $\Omega$ and $p_\omega$;
2. Not knowing the loss operator;
3. Not knowing the form and parameters of the aleatory process;
4. Not knowing which future will prevail (scenarios);
5. Not knowing the footprint function.

Two crucial tools:

- Adding a ‘margin for error’ (ubiquitous);
- Sensitivity analysis (very under-used).
PE curve uncertainty from sampling, $\alpha = 5\%$
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- **N = 100**
  - Graph showing area of damage (sq miles) vs. uncertainty.
  - The shaded area represents the 5% confidence interval.

- **N = 500**
  - Similar graph to N = 100, but with a larger sample size.
  - The shaded area is wider, indicating increased confidence.

- **N = 1000**
  - Graph showing area of damage (sq miles) vs. uncertainty with the largest sample size.
  - The shaded area is the widest, indicating the highest confidence.

The graphs illustrate how the confidence in the PE curve increases with larger sample sizes, with the 5% confidence interval being shaded.
PE curve uncertainty from sampling, $\alpha = 5\%$
Uncertainty about the loss operator can be integrated out

Previously we had

$$\ell_\omega = \sum_{x \in X} \text{area}(x) 1\{v_\omega(x) > 1\}.$$  

But it is straightforward to generalise to

$$\ell_\omega = E\left\{ \sum_{x \in X} \text{area}(x) 1\{\text{damage in } x\} \mid \omega \right\}$$

$$= \sum_{x \in X} \text{area}(x) \Pr \{\text{damage in } x \mid \omega\}$$

$$= \sum_{x \in X} \text{area}(x) \{0 \vee g(v_\omega(x)) \wedge 1\}$$

for some specified non-decreasing $g(\cdot)$.  

This changes the shape of the PE curve but it does not make the PE curve uncertain.
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Multiple future scenarios

We often represent the future in terms of scenarios. For earthquakes these would be socio-economic scenarios that affect population density, building standards and quality, resilience. What prevents us from integrating out future uncertainty?
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Prob. of exceedence (PE)

Area of damage (sq miles)

PE curve for single scenario
Multiple future scenarios

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![Graph](image-url)

Different scenarios, each PE curve is conditional on the scenario.
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Prob. of exceedence (PE)

Area of damage (sq miles)

Probability
- Scen. A, 30%
- Scen. B, 20%
- Scen. C, 20%
- Scen. D, 20%
- Scen. E, 10%
- Unconditional

Unconditional PE curve: this step is quite contentious!
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**Prob. of exceedence (PE)**

Unconditional PE curve: this step is quite contentious!

Area of damage (sq miles)

Probability
- Red: Scen. A, 30%
- Green: Scen. B, 20%
- Blue: Scen. C, 20%
- Cyan: Scen. D, 20%
- Pink: Scen. E, 10%
- Black: Unconditional
Other sources of epistemic uncertainty

Not knowing the form and parameters of the aleatory process.
This is standard statistics: challenging, but not obscure.

Not knowing the footprint function.
This is indeed hard, but progress is being made in the statistical field of Computer Experiments, notably in the MUCM project.
Summary

‘Aleatory’ uncertainty
The inherent uncertainty in a natural hazard. A PE curve is a summary of aleatory uncertainty as represented in terms of loss.

‘Epistemic’ uncertainty
‘Other’ uncertainty, notably that which arises from our incomplete knowledge. Things to remember:

1. Limited sampling introduces uncertainty, and can be represented as confidence bands on the PE curve, e.g. when estimating quantiles.

2. Some epistemic uncertainties can be integrated out. The PE curve does not become ‘uncertain’, it becomes conditional.

3. Others are more challenging, notably multiple future scenarios, and accounting for footprint function limitations.