How can probability be used to quantify uncertainty in climate predictions?

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Interpretations of probability

The axioms of probability theory are not in dispute: (0) There exists a set S; (1) For every $A \subset S$, $\Pr(A) \geq 0$; (2) $\Pr(S) = 1$; (3) if A and B are disjoint, then $\Pr(A \cup B) = \Pr(A) + \Pr(B)$.

But these axioms do not tell us how probability theory is relevant in our lives.

Classical Early treatment linked to gambling, applies to 'equally-likely outcomes' (e.g., card hands).

Frequentist Probability $=$ limiting relative frequency in a large collection of 'identical trials'.

Logical (or Necessary) Probability is the extension of two-valued logical to uncertainty.

Bayesian (or Subjective, or Personalistic) Probability is a way of quantifying an individual's uncertainty.

Two features of the Bayesian approach

- The Bayesian approach is based on an operational definition of probability.

  > If I assert that $\Pr(A \mid B) = p$, then I am indicating that p is the value I would select for x when facing the penalty
  >
  > $$I_B(I_A - x)^2$$
  >
  > where I_A and I_B are indicator functions of A and B. $\Pr(A)$ is shorthand for $\Pr(A \mid S)$, where necessarily $I_S = 1$.

- The axioms of probability then follow as theorems from the coherence condition that I would never willingly specify probabilities $\boldsymbol{p}$ if there was another choice for which the penalty was never larger and sometimes smaller.

Probabilities make things clearer

Let S be the sea-level rise at Margate in 2100, in metres. The Mayor would like to know $\Pr(S \geq 2 \mid \text{obs. data})$.

The (Bayesian) Guru approach

— *After thinking deeply and taking account of all available information, my probability is 0.5.*

— *Thank you oh Guru, you are wise indeed.*

— *No problem, that will be One Meellion Dollars.*

Probabilities make things clearer (cont)

A more transparent approach

Suppose the main source of uncertainty is the behaviour of ice, for which we have two competing models.

1. $\Pr(S \geq 2 \mid \text{obs. data}) =$

$$\sum_{i=1,2} \underbrace{\Pr(S \geq 2 \mid \text{ice model } i)}_{\text{Not too hard?}} \underbrace{\Pr(\text{ice model } i \mid \text{obs. data})}_{\text{Tricky}} .$$

2. $\Pr(\text{ice model } i \mid \text{obs. data}) \propto$

$$\underbrace{\Pr(\text{obs. data} \mid \text{ice model } i)}_{\text{Not too hard?}} \underbrace{\Pr(\text{ice model } i)}_{\text{Not so tricky? Less important?}} \quad i = 1, 2.$$

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Do we have to use probabilities?

- We may feel uncomfortable with Bayesian probabilities, but decisions still have to be made.
- Relocating my business: costs for outcomes and decisions.

What melts?	*A*. Nothing	*B*. G'land	*C*. W.Ant. ($\Rightarrow$ B)	*D*. E.Ant. ($\Rightarrow$ C)
Stay in London	0	110	160	230
Move to Geneva	5	15	65	135
Move to Lhasa	30	40	60	130

- Minimax does not require probabilities. The solution is to move to Lhasa, because the outcome from (D) is least-bad there.
- But if $\Pr(D)$ is small then this seems like an over-reaction. Bayesian decision theory with $p = (0.4, 0.4, 0.15, 0.05)$ would suggest relocating to Geneva.

Next-generation modelling

The notion that some of our model's parameters are uncertain is now well-established, and the motivation behind ensemble experiments.

- The next step for model-building:

  Old-school: probs bolted on as an after-thought. $\longrightarrow$ Next-Gen: Probs an integral part of the modelling.

- For example, suppose there are two competing ice models. Our judgement might previously have been

  — *Ice model 1 is probably better than model 2, so we'll stick 1 in our climate model.*

  But we could say

  — *Ice model 1 is probably better than model 2, so we'll stick them both in (with an extra parameter to select between them), but assign* $Pr(model\,1) = 0.6$ *and* $Pr(model\,2) = 0.4$.

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Stochastic forcing / stochastic models

We can think of our model as $f(\theta, x)$ where $\theta =$ model parameters and $x =$ forcing, e.g., atmospheric CO2 from 1850 to today. Ensemble experiments vary θ. But what about uncertainty in x?

- Current approaches tend to use $g(\theta) \triangleq f(\theta, \mathrm{E}_x(x))$, but what we really want is $g'(\theta) \triangleq \mathrm{E}_x(f(\theta, x))$.
- One simple solution is to use Monte Carlo integration, to compute

$$g'(\theta) \approx \frac{1}{k} \sum_{i=1}^{k} f(\theta, x_i) \quad x_i \overset{\text{iid}}{\sim} F_x$$

  which is stochastic forcing ($k = 1$ is an extreme case).
- For $k \geq 2$, statisticians have developed powerful variance reduction methods for which $x_i \overset{\text{iid}}{\nsim} F_x$, but as a consequence we can get a better estimate of $g'(\theta)$ ... Should we build these into the models?

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Summary

- Except in a few easily identified cases, probabilities in climate prediction are Bayesian; they are subjective descriptions of uncertainty.

There is no THE probability!

- The probability calculus makes it easier for us to specify probabilities for complicated propositions. Usually, incorporating probabilities gives more sensible decisions.
- Modellers can make more nuanced judgements about model-structure. Certain types of uncertainty can be 'built in'.

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