

HW4, Theory of Inference 2015/6

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1. Define what it means for a decision rule to be admissible, and illustrate with a diagram containing both admissible and inadmissible decision rules. State Wald's Complete Class Theorem. Explain why an inadmissible decision rule is not necessarily dominated by a Bayes rule. (Hint: a diagram similar to that used in the proof of Complete Class Theorem might help.) [15 marks]

2. Give a simple direct proof that all Bayes Rules with strictly positive prior probabilities are admissible (hint: use proof by contradiction). [10 marks]

3. In the case of point estimation, describe the behaviour of the Bayes Rule for the loss function

$$L(\tilde{\theta}, \theta) = 1 - \mathbf{1}_{|\tilde{\theta} - \theta| \leq \epsilon}$$

in the limit as $\epsilon \downarrow 0$. For convenience, consider the case where $\Omega = \mathbb{R}$; you might find a diagram helpful. [10 marks]

4. In the case of hypothesis testing, show that if the loss function is zero-one then the Bayes Rule is to select the hypothesis with the largest posterior probability. [5 marks]

5. An analyst at a hospital has 'failed to reject' the hypothesis that mortality rates from emergency surgery are the same on weekdays and weekend-days. He claims that this proves that there is no excess mortality at the weekend. What do you make of his claim? [15 marks]