HW5, Theory of Inference 2015/6

Jonathan Rougier
School of Mathematics
University of Bristol UK

In these questions, we have $Y \sim \{f, \Omega\}$ as usual. The questions without marks are unlikely to turn up on an exam, but you should do them anyway, for practice.

1. Let X be a random quantity with distribution function F_X. Prove that if F_X is continuous then $Y := F_X(X)$ has a Uniform distribution. Note that this is a more general result than the easy proof in the case where F_X is strictly increasing. See Casella & Berger (2002, Thm 2.1.0).

2. This standard result is used in the Marginalisation theorem for confidence procedures. Let $q(y)$ and $r(y)$ be first-order sentences (i.e. statements about y which are either true or false). Prove that if

$$\forall(y)(q(y) \to r(y)),$$

then $\Pr\{q(Y); \theta\} \leq \Pr\{r(Y); \theta\}$ for all $\theta \in \Omega$.

3. Consider functions $C : \mathcal{Y} \times [0,1] \times [0,1] \to 2^{\Omega}$ of the form

$$C(y, u, \alpha) = \big\{\theta_j \in \Omega : a \cdot u + b > \alpha\big\}.$$

Suppose that $U \sim \mathrm{U}[0,1]$, independently of Y for all $\theta \in \Omega$. State the model for (Y, U). Under what conditions on a and b is C a family of level $(1-\alpha)$ confidence procedures for θ? Interpret this result as a reflection on the usefulness of confidence procedures. [15 marks]

4. Using Wilks's Theorem, construct a family of approximately exact confidence procedures which does not satisfy the Level Set Property. Sketch a level 95% confidence set from this family in the case where $\Omega \subset \mathbb{R}$. [10 marks]

5. Prove that confidence sets which satisfy the Level Set Property are 'transformation invariant'. Start by exploring what this might mean (hint: think about expressing the same model with a different parameter ϕ, where $g : \theta \mapsto \phi$ is bijective). [15 marks]