HW4, Theory of Inference 2016/7

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In the following questions, I show marks in square brackets, to give you an idea of the approximate tariff the question would carry in an exam.

1. We proved the Complete Class Theorem for the special case where Ω is finite. Prove the 'if' branch without this constraint. I.e., prove that a decision rule δ is admissible if it is a Bayes rule for some positive prior distribution π. Hint: use proof by contradiction. [10 marks]

2. Consider the model $\mathcal{E} = \{y \in \mathbb{N}, \lambda \in \mathbb{R}_{++}, f\}$, where f is the Poisson probability mass function (and $\mathbb{N} = \{0, 1, \dots\}$). Let the prior for Λ be Gamma(a, b), where a is the 'shape' parameter and b is the 'rate' parameter.[1] Derive the Bayes rule for the point estimate for Λ under quadratic loss. [10 marks]

   In an exam you would be given the Poisson PMF and the Gamma PDF.

3. For the loss function given in eq. (3.7a) in the handout, confirm the Bayes rule for the two extreme cases $\kappa \downarrow 0$ and $\kappa \to \infty$. [10 marks]

4. Prove the result in eq. (3.8) in the handout. [10 marks]

5. Prove that for hypothesis testing, the Bayes rule for the zero-one loss function is to select the hypothesis with the largest posterior probability. [10 marks]

6. Consider a 2D parameter space partitioned into two non-degenerate hypotheses, $\Omega = \Omega_0 \cup \Omega_1$. On a single figure, sketch three set estimates: $\delta_1(y^{\text{obs}})$ accepts H_0, $\delta_2(y^{\text{obs}})$ rejects H_0, and $\delta_3(y^{\text{obs}})$ is undecided about H_0. Redraw your picture for the case where Ω_0 is composite degenerate. [10 marks]

[1] Remember that 'Λ' is capital 'λ'.