HW6, Theory of Inference 2016/7

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In the following questions, I show marks in square brackets, to give you an idea of the approximate tariff the question would carry in an exam. In these questions, the model is the usual $\{\mathcal{Y}, \Omega, f\}$.

1. Define a 'family of confidence procedures'. What additional property do you think that 'nesting' families of confidence procedures have? [5 marks]

2. Define a 'family of significance procedures'. Prove that if p is a family of significance procedures, then $\sup_{\theta\in\Omega_0} p(y;\theta)$ is a significance procedure for the null hypothesis $\Omega_0 \subset \Omega$. [10 marks]

3. **Theorem 1** (The Duality Theorem)**.** *Let p be a family of signficance procedures. Then*

$$C(y;\alpha) = \big\{\theta \in \Omega : p(y;\theta) > \alpha\big\}$$

is a nesting family of confidence procedures. Conversely, let C be a nesting family of confidence procedures. Then

$$p(y;\theta) = \inf\big\{\alpha : \theta \notin C(y;\alpha)\big\}$$

is a family of significance procedures.

Prove the first half of this theorem. [10 marks]

4. We proved in the lecture that

$$p(y;\theta) = \Pr\{g(Y) \geq g(y);\theta\}$$

is a family of significance procedures, for any $g : \mathcal{Y} \to \mathbb{R}$. Prove this result directly in the special case of $g(y) = c$, where c is any constant. [10 marks]

5. If your p-value is small, then the observations are improbable under your null hypothesis. What information do you need to compute the probability that your null hypothesis is true, given the observations? [10 marks]