Of Donkeys and Nomograms

Kate Milner
Beechwood Veterinary Centre (Active Vet Care)
Woodley, Berkshire

Jonathan Rougier*
Department of Mathematics
University of Bristol

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1 Introduction

If you are a vet, you will often need to know the weight of an animal, for example to prescribe the right amount of a drug. If your animal is a rabbit, this is not a problem. But if your animal is large, say a donkey, and if you are out in the field, then you will not be able to weigh the animal directly. However, if you have a tape measure then you can weigh the animal indirectly, if you have a tool for converting tape measurements into weights. In this article we describe how we constructed such a tool—a nomogram—for Kenyan donkeys.

Nomograms have been used before to predict weight on the basis of simpler measurements, including for horses, mules, and donkeys (see, e.g., Eley and French, 1993; Carroll and Huntington, 1988; Pearson and Ouassat, 1996; Kay *et al.*, 2004). What we present here is a 'more statistical' treatment, which we hope can serve as a template for other similar studies. We consider a richer set of possible models, an appropriate loss function for choosing between them, the constraints of practical usage, and a careful assessment of accuracy. Our analysis was performed in the statistical computing environment R (R Core Team, 2013). All of our code and data is available on-line: see the Resources section at the end of the article.

*Corresponding author: email `j.c.rougier@bristol.ac.uk`.

Figure 1: Kenyan donkeys, descended from the Nubian wild ass and the Somali wild ass (photos: Kate Milner)

2 Kenyan donkeys

In 2010, The Donkey Sanctuary, a UK registered charity based in Sidmouth, Devon, funded one of us (Kate) to travel to Kenya. The purpose of the trip was to assemble a dataset and to construct a parallel-scale nomogram (see Section 3) for predicting the weight of Kenyan donkeys according to their other more accessible measurements, listed below.

The current population of donkeys in Kenya is estimated to be about 1.8 million. The predominant breeds are descendants and crosses of the Nubian wild ass (Equus africanus africanus) and the Somali wild ass (Equus africanus africanus somaliensis); see Figure 1. Kenya is an agricultural country and donkeys are important for transporting goods, such as crops, water and building materials. In some regions, such as the island Lamu, donkeys are also important for transporting people. Less frequently, they are used for ploughing.

Data for 544 donkeys were collected at seventeen different sites located in the regions surrounding Yatta district in Eastern province and Naivasha district in the Rift Valley province, Kenya, during the period from 23 July to 11 August 2010. The predominant use of donkeys in the Yatta district is as pack donkeys, whereas in the Naivasha district they are mainly used to pull carts. The donkeys were brought to the sites for de-worming by The Donkey Sanctuary. Where possible, all presented donkeys were included in the study, excluding those that were pregnant or had visible disease. Where that was too many to assess, a sample was used.

Four measurements were made for each donkey: liveweight (kg), heart girth

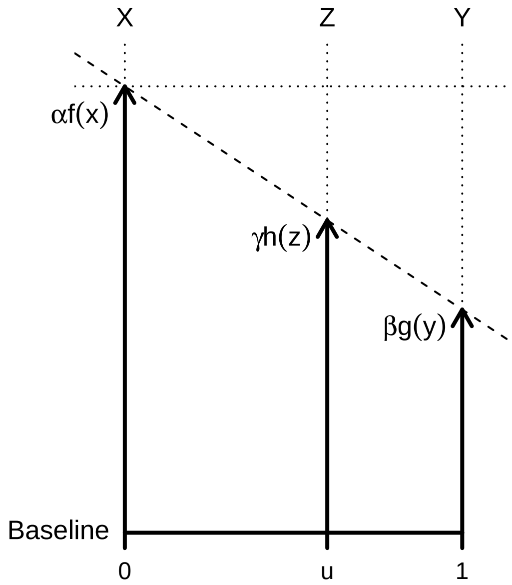

Figure 2: Geometry of a parallel-scale nomogram, after Fig. 2 in Doerfler (2009).

(cm), height (cm), and length (cm); more details about these and the following measurements are given in the Appendix. Each donkey's body condition score (BCS), age, and sex were also recorded. The BCS is an ordinal scale running from 1 (emaciated) through 3 (healthy) to 5 (obese), including half scores. Age in years was assessed from incisors into the categories <2, 2–5, 5–10, 10–15, 15–20, and >20. Sex was 'stallion', 'gelding', or 'female'.

3 Parallel-scale nomograms

This section is self-contained—we return to the donkeys in section 4. Suppose that three quantities x, y, and z are related in the form

$$f(x) + g(y) = h(z) \tag{1}$$

for specified monotonic functions f, g, and h. In this case it is possible to represent the relationship pictorially as a parallel-scale nomogram. Doerfler (2009) provides an excellent review of nomograms, from which the following explanation is taken.

In a parallel-scale nomogram there is a vertical axis for each quantity, and a straight edge connecting values on any two axes intersects the correct value on the third. Figure 6 is an example of a parallel-scale nomogram. The geometrical construction of such a nomogram is shown in Figure 2. The four unknowns are α, β, γ, and u. By similar triangles,

$$\frac{\alpha f(x) - \gamma h(z)}{u} = \alpha f(x) - \beta g(y)$$

which gives, on rearranging,

$$\alpha(1-u)f(x) + \beta u g(y) = \gamma h(z). \tag{2}$$

Now suppose that (2) must hold for all x, y, and z which satisfy (1). This implies that

$$\alpha(1-u) = \beta u = \gamma,$$

which gives two equalities for four unknowns. Hence we have a free choice of, say, α and β, and then

$$u = \frac{\alpha}{\alpha+\beta} \quad \text{and} \quad \gamma = \frac{\alpha\beta}{\alpha+\beta}.$$

In practice, we might provide lower and upper limits for x and y, which determine α and β. The lower limits of x and y determine the vertical location of the bottom of the z-axis, whose horizontal location is u and whose scale is γ.

Parallel-scale nomograms are extremely easy to use, including in fieldwork. For example, a vet might make two measurements, and mark these as crosses on two of the axes—it does not matter if her hands are wet or dirty. She might then join the crosses with a freehand straight line, or a ruled line if she has a straight edge handy—it would be sensible to ensure that the two outer axes are not more than a pencil-length apart. The nomograms could be made available as a pad of disposable sheets, or as a single reusable laminated sheet. An underrated practical feature of nomograms is that they are invariant to changes in the aspect ratio, which might happen when the nomogram is printed or photocopied.

There are more complicated nomograms than parallel-scale nomograms, discussed in Doerfler (2009): some of these are very beautiful, and the mathematics is intriguing. While these allow for richer relationships, possibly with more than three quantities, they are also harder to use. We will stick with parallel-scale nomograms.

4 Back to donkeys

4.1 Implementing the nomogram

A donkey is basically an elliptical cylinder with appendages. So we expect its weight to be approximately proportional to $\mathsf{Girth}^2 \times \mathsf{Length}$. It is possible that a donkey's less-cylindrical aspects could be accommodated by also including Height as an additional predictor; however, this cannot be represented in a parallel-scale nomogram. Therefore our starting point is the model

$$\underbrace{a + b \cdot \log(\mathsf{Girth})}_{f(\mathsf{Girth})} + \underbrace{c \cdot \log(\mathsf{Length})}_{g(\mathsf{Length})} = h(\mathsf{Weight}).$$

But it is an empirical question whether we might do better replacing Length with Height. Although it seems natural to use a logarithm for h, we allow ourselves more flexibility by using the Box-Cox power transformation:

$$h(z;\lambda) := \begin{cases} \dfrac{z^\lambda - 1}{\lambda} & \lambda \neq 0 \\ \log(z) & \lambda = 0, \end{cases}$$

where λ is to be determined.

We also have the possibility of adjustments for discrete factors, namely BCS, Age and Sex. Adjustments such as "Add 5 kg for a gelding" are simple enough to be expressed in the rubrick of the nomogram. Interactions, on the other hand, such as "Subtract 5 kg for an animal which is both a gelding and 5–10 yo" are more prone to error in the field, and we will avoid them. For the same reason, we favour additive adjustments in units of kilograms, rather than proportionate adjustments in units of percent, even though the latter might be more plausible, physiologically.

Incorporating additive adjustments, the 'nomogram + factors' prediction has the form

$$\mathsf{Weight} = \text{nomogram}(\mathsf{Girth}, \mathsf{Length}) + \beta_{\mathsf{gelding}} \mathbb{1}_{\mathsf{Sex=gelding}} + \cdots$$

where $\mathbb{1}$ is the indicator function. We will fit our models using least-squares regression. This means we have to convert the regression coefficients estimated on the h(kg) scale to β's on a kg scale. Each adjustment is specified by the level of a factor, such as level gelding of factor Sex, and we use

$$\beta_{\textsf{gelding}} := n_{\textsf{gelding}}^{-1} \sum_{i \in \textsf{gelding}} \left\{ h^{-1}\big(\hat{h}_i\big) - h^{-1}\big(\hat{h}_i(\textsf{stallion})\big) \right\}$$

where $\hat{h}_i$ is the least-squares prediction for the ith donkey on the h(kg) scale, and stallion is the reference level for Sex. Here $\hat{h}_i(\textsf{stallion})$ is the predicted weight of a hypothetical donkey which is just like donkey i in every respect, except for being a stallion instead of a gelding. If h was the identity function, then $\beta_{\textsf{gelding}}$ would be the regression coefficient on $\mathbb{1}_{\textsf{Sex}_i = \textsf{gelding}}$.

4.2 Model selection

We now have a set of possible models: Length versus Height as the second predictor, and a range of values for the Box-Cox parameter λ.

Which do we prefer? Consider the loss function, from the point of view of the donkey's health. This loss function depends on the drug being prescribed. For drugs like wormers and antibiotics the therapeutic window is quite wide, and it is better to overdose because otherwise the infestation/infection might not be treated, and an underdose might lead to drug resistance. For drugs like anaesthetics and analgesics the therapeutic window is narrower, and it is better to underdose because the effect can be observed, and the dose can be adjusted. So we actually have two loss functions: ideally our preferred model would be the best model under both of them. Figure 3 shows the two loss functions we use; these are exponentially-tilted quadratics.

Note that we have defined the relative error as 'actual / predicted'. This is because the value available to the vet is the donkey's predicted weight, not the actual weight, and the natural question for her to ask is "How different is this donkey's actual weight from its predicted weight of 175 kg?" (say). Thus a relative error of -10% indicates that the actual weight is 10% smaller than predicted, and hence the risk is of overdosing, not underdosing. For effective treatment it is crucial that we provide a reliable assessment of our tool's accuracy, uncontaminated by our data-driven modelling decisions. Therefore we set aside every fifth case in our dataset after ordering by weight, to be used purely to assess accuracy.

Proceeding with the remain four fifths of our dataset, Figure 4 shows the sample mean loss values for Length versus Height, for different values of λ, and

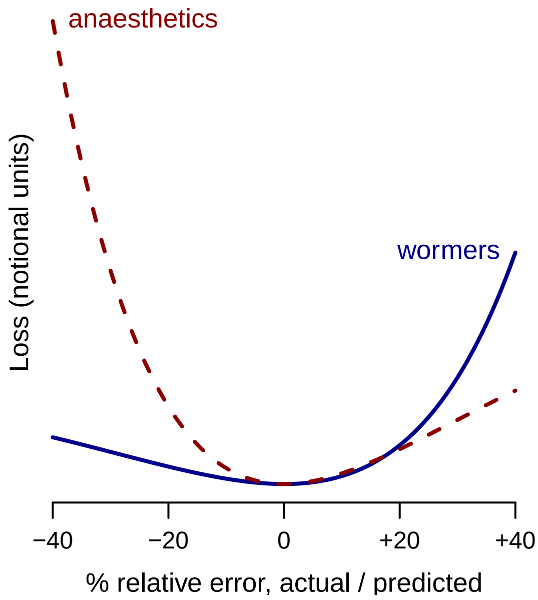

Figure 3: Two loss functions for predicting a donkey's weight. The blue line represents wormers and antibiotics, and the red dashed line represents anaesthetics and analgesics. Note that a negative relative error corresponds to an overdose.

for the two loss functions. These are computed using a full set of β's for all of the levels of BCS, Age, and Sex. For both loss functions, Length beats Height, as we anticipated. Looking at Length, the optimal value for λ seems to be $\lambda^* = 0.5$, or

$$h(\mathsf{Weight}) = 2(\sqrt{\mathsf{Weight}} - 1). \tag{3}$$

We will adopt this from now on. We did, however, also check the results for $h =$ 'log' (i.e. $\lambda = 0$), which is the standard choice, and there was no discernible difference. As $\log(x)$ is nearly linear in $\sqrt{x}$ over the range of donkeys' weights, this is not surprising.

Now we turn to the additive adjustments. We are looking to remove factors, and recode the levels of those that remain, to reduce the cognitive burden of our tool. The estimated β's are shown in Figure 5. Clearly Sex can be removed, but BCS and Age are both important. We recode Age as the three levels $<$2, 2–5, and $>$5, which is physiologically plausible. The clearly differentiated values for BCS suggest that the qualitative scale is well-defined. Possibly we could merge BCS levels 2 and 2.5, but the saving would be minimal.

We refit the model with these recoded factors, taking the most populous

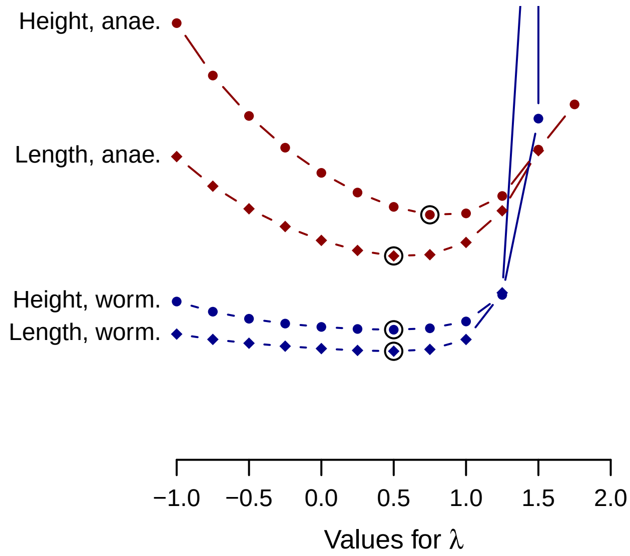

Figure 4: Sample mean loss values for the two loss functions in Figure 3, for Length versus Height as the second quantity, and for a range of values of the Box-Cox parameter λ. The minimum values are circled.

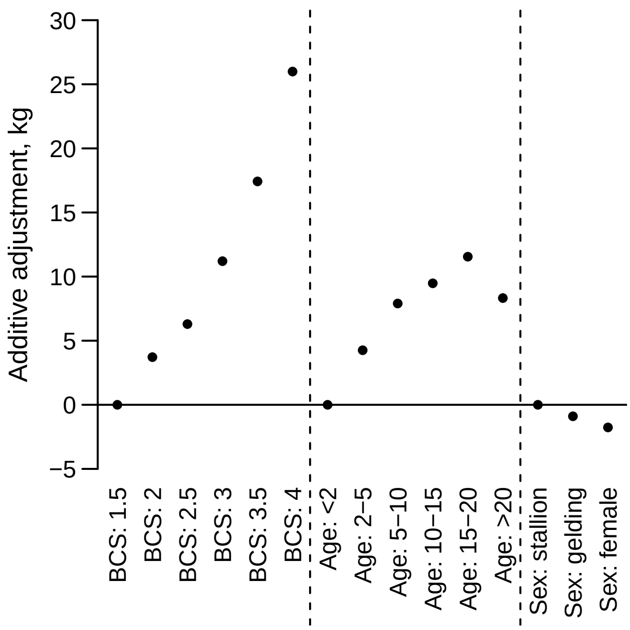

Figure 5: The estimated additive adjustments for the factors, with Length as the second quantity, and $\lambda = 0.5$.

Table 1: Additive adjustments for factors at non-reference levels, in kilograms.

Factor			
BCS		Age	
1.5	−10	<2	−8
2	−6	2–5	−4
2.5	−5	>5	none
3	none		
3.5	+6		
4	+14		

Table 2: Distribution of relative errors of our tool in the holdout sample of 108 donkeys.

	Relative error, actual / predicted				
	< −10%	−10% to 0%	0% to +10%	+10% to +20%	> +20%
Proportion	8%	44%	44%	3%	1%

levels as the reference (BCS = 3 and Age = >5). Our resulting model is (4sf)

$$f(\mathsf{Girth}) = -107.0 + 19.91 \cdot \log(\mathsf{Girth})$$
$$g(\mathsf{Length}) = 7.712 \cdot \log(\mathsf{Length})$$

plus h defined in (3). The nomogram is shown in Figure 6. Readers can confirm from the nomogram that the predicted weight of a donkey with BCS = 3 and Age = >5 who has Girth = 122 cm and Length = 103 cm is 175 kg. This corresponds to donkey number 78 in our dataset, whose actual weight is 183 kg, for a relative error of about +5%. The additive adjustments for those donkeys with factors at non-reference levels are given in Table 1, rounded to the nearest kilogram.

Finally, we assess our tool's accuracy, using the hold-out sample. The prediction of weight proceeds exactly as if in the field; i.e. using only the information in Figure 6 and Table 1. Figure 7 and Table 2 show that it is reasonable to claim that the typical accuracy of our tool is about ±10%, and that this is relatively consistent over the range of predicted weights from 75 kg to 200 kg.

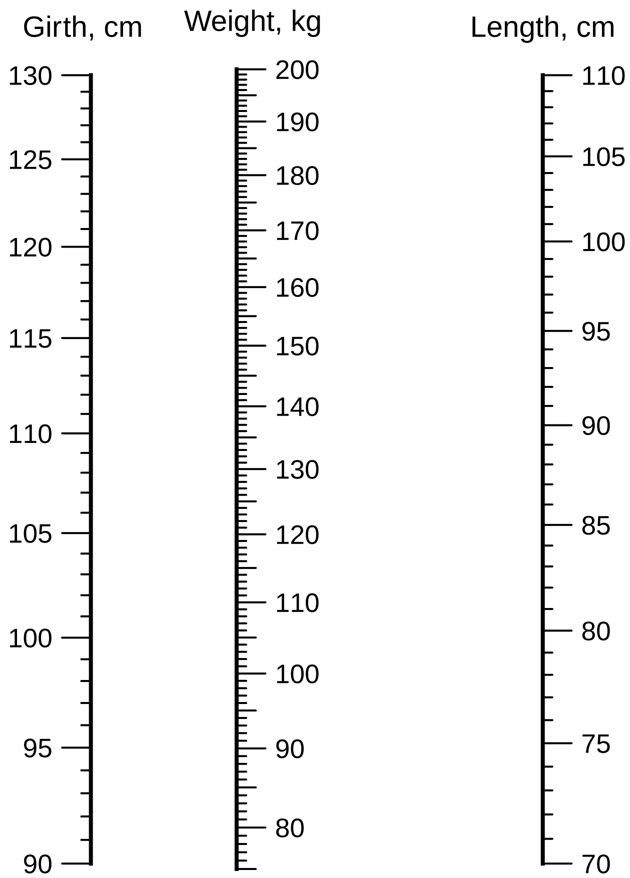

Figure 6: Nomogram for Kenyan donkeys with BCS = 3 and Age = >5. To predict Weight, join the Girth and Length values with a straight line.

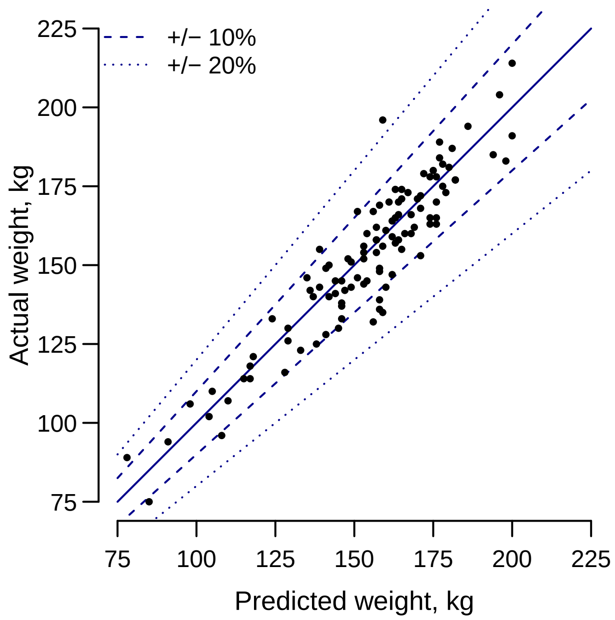

Figure 7: Hold-out sample of 108 donkeys. Predicted weight versus actual weight, with relative error bands.

Appendix: more details on the measurements

Donkeys were weighed on an electrical weighing platform (Salter Brecknell PS-1000 scale version 1.0). Accuracy was checked with a standard 6 kg weight. Weights were recorded to the nearest kilogram. To check repeatability, 31 donkeys were weighed twice, with other donkeys being weighed between the two measurements; no weights varied by more than 1 kg. Heart girth: circumference from caudal edge of withers and behind the forelimb, around the girth, using a measuring tape. Height: distance from ground level to highest point of withers measured using a measuring stick. Length: distance from olecranon (point of elbow) to tuber ischii (pin bone) using a measuring tape. See the Resources for a useful guide on these measurements and also the BCS.

Donkeys were de-wormed and marked with a crayon immediately following data collection to avoid them being recorded for a second time.

Three of the 544 donkeys were excluded from the statistical analysis as being unrepresentative: one was a baby, one had a BCS of 1, and one had a BCS of 4.5; we dropped these two levels from the BCS factor. These three donkeys were easily identified using a parallel coordinates plot (`parcoord` in the `MASS` package).

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Resources

- R package for parallel-axis nomograms, `http://www.maths.bris.ac.uk/~mazjcr/paranomo_1.0.tar.gz`
- R package containing the data, `notdoneyet`
- *The Lost Art of Nomography* by Ron Doerfler, `http://myreckonings.com/wordpress/wp-content/uploads/JournalArticle/The_Lost_Art_of_Nomography.pdf`
- The Donkey Sanctuary, `http://www.thedonkeysanctuary.org.uk/`. The charity's book *The Complete Book of the Donkey* (author Elisabeth D. Svendsen, 2009, Kenilworth Press) contains lots of information about donkeys, and many beautiful photographs (of donkeys).
- *Monitoring your donkey's weight and condition*, `http://www.thedonkeysanctuary.ie/files/ireland/Weight-Management-And-Condition-Scoring.pdf`

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